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EIGENELEMENTS OF THE FINITE FOURIER TRANSFORM AND THEIR APPLICATION TO ANTENNA PATTERN SYNTHESIS

THOMAS S. FONG

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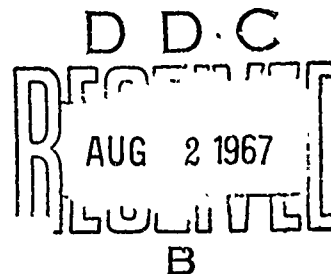
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AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

ABSTRACT

The eigenvalues and eigenfunctions of the finite Fourier transform corresponding to parameter $c = 10\pi$, 15π , 20π , and 25π are computed and tabulated. The computational details are given and discussed. An antenna pattern synthesis technique that makes use of these eigenelements is illustrated by examples. When the super-directive ratio is constrained to reasonable values, the number of eigenfunctions that contribute significantly to the pattern is found to be approximately equal to twice the number of wavelengths in the aperture. A pattern synthesized with the above eigenfunctions is compared with the pattern obtained by Woodward's method, and considerable improvement is noted over that method.

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1.0 INTRODUCTION

Eigenfunctions of the finite Fourier transform, often referred to in the literature as angular prolate spheroidal wave functions of the first kind, have received considerable attention (Flammer, 1957; Stratton, 1956). These functions and the corresponding eigenvalues occupied an important role in a recently studied technique for the synthesis of antenna radiation patterns (Rhodes, 1963; Fong, 1965). The range of tabulated values for both the eigenfunctions and the eigenvalues has been so limited, however, as to restrict any practical application of the synthesis technique until additional values were available. A few usable sets of eigenfunctions and eigenvalues, suitable for antenna aperture sizes of 10, 15, 20, and 25 wavelengths, were computed as a basis for the present study and then applied to the synthesis of patterns from apertures of the same size.

An extensive table of values has not been presented nor has mathematical completeness been emphasized since the objective is the computation of the eigenelements; only the essential steps are included in the discussion. The notation of the parameters used and the physical meanings are consistent with those of the earlier report (Fong, 1965) and are repeated here for convenience.

L = half the aperture length

c = $2\pi L/\lambda$

u = $\sin \theta$, $\theta = 0$ at broadside

x = ξ/L , ξ is the coordinate along the aperture

λ_n = the n^{th} eigenvalue

ψ_n = the n^{th} eigenfunction

The normalization has been taken such that

$$\int_{-1}^1 \psi_n^2(x) dx = 1$$

with $\psi_n(0) > 0$ for n even and $\psi'_n(0) > 0$ for n odd.

2.0 TECHNICAL DISCUSSION

2.1 THE DIFFERENTIAL EQUATION SATISFIED BY THE EIGENFUNCTIONS

A direct computation of the eigenelements of the finite Fourier transform

$$\int_{-1}^1 \psi(x) e^{jcu x} dx = \lambda \psi(u)$$

as outlined in an earlier report (Fong, 1965) was initially considered. In this approach the kernel is expressed in the form

$$\sum_{m,n=0}^{\infty} A_{mn} \varphi_m(x) \varphi_n(u)$$

where $\{\varphi_p\}$, $p = 0, 1, \dots$, is a set of known basis. For computational purposes the kernel is approximated by a finite sum, and the integral equation reduces to a finite set of simultaneous equations or a matrix equation of the form $Ae = \lambda e$. Because of the essential equality of the 0th, 4th, 8th, ..., M^{th} eigenvalues, where M depends on the parameter c or the aperture length, the first five digits being the same, the finite sum approximation and the error in computing the coefficients A_{mn} above by integration present considerable uncertainty on the computed eigenvectors e_0, e_4, \dots, e_M from which the eigenfunctions $\psi_0, \psi_4, \dots, \psi_M$ are computed. The same difficulty occurs in finding e_1, e_5, \dots, e_{M+1} , etc.

The uncertainty in the eigenvectors may be seen from the following consideration. If it is supposed that λ_0, λ_4 , and λ_8 are nearly the same, then $A(\alpha e_0 + \beta e_4 + \delta e_8) \approx \lambda_0(\alpha e_0 + \beta e_4 + \delta e_8)$, where α, β , and δ are constants. This relationship implies that $\alpha e_0 + \beta e_4 + \delta e_8$ is an eigenvector associated with λ_0 , approximately. In fact, any linear combination of e_0, e_4 , and e_8 would satisfy the equation $Ae = \lambda e$ approximately. Consequently, within the limits of the accuracy of computation,

erroneous solutions can be generated that correspond to arbitrary linear combinations of the correct eigenfunctions. To avoid this difficulty, the number of terms in the finite approximation must be quite large, say at least 100 terms, in addition to the requirement that the coefficients A_{mn} must be accurately determined. The computing time becomes quite long, although the technique is straightforward. In view of this fact, an alternative approach that considered an equivalent Sturm-Liouville differential operator was examined.

For completeness of this discussion, some of the development appearing in the earlier studies, Rhodes (1963) and Fong (1965) is repeated here. The relationship is established between the eigenelements of the integral operator with kernel e^{jcux} above and the eigenelements of the differential operator

$$L_x = \frac{d}{dx} (1 - x^2) \frac{d}{dx} - c^2 x^2$$

It is supposed that u and v are two continuous, twice differentiable functions. Then

$$\begin{aligned} \int_{-1}^1 (u L_x v - v L_x u) dx &= \int_{-1}^1 u \left\{ \frac{d}{dx} \left[(1 - x^2) \frac{dv}{dx} \right] - c^2 x^2 v \right\} dx \\ &\quad - \int_{-1}^1 v \left\{ \frac{d}{dx} \left[(1 - x^2) \frac{du}{dx} \right] - c^2 x^2 u \right\} dx \\ &= (1 - x^2) \left(u \frac{dv}{dx} - v \frac{du}{dx} \right) \Big|_{-1}^1 \\ &= 0 \end{aligned} \tag{1}$$

It follows from straightforward differentiation that

$$L_x e^{jcux} = L_u e^{jcux} \tag{2}$$

Let $\theta(x)$ be a solution to

$$L_x \theta + k \theta = 0 \quad (3)$$

That is, θ and k are, respectively, an eigenfunction and an eigenvalue of L_x . Denote

$$\int_{-1}^1 e^{jcux} \theta(x) dx$$

by $\psi(u)$. Consider $L_u \psi(u)$.

$$L_u \psi(u) = \int_{-1}^1 L_u e^{jcux} \theta(x) dx$$

As a consequence of Equations (1), (2), and (3) above, it develops that

$$\begin{aligned} L_u \psi(u) &= \int_{-1}^1 \theta(x) L_x e^{jcux} dx \\ &= \int_{-1}^1 e^{jcux} L_x \theta(x) dx \\ &= -k \int_{-1}^1 e^{jcux} \theta(x) dx \\ &= -k \psi(u) \end{aligned} \quad (4)$$

In view of these Expressions (4), it may be concluded that $\psi(x)$ and $\theta(x)$ are proportional, or the eigenfunctions of the differential operator and the integral operator are the same. There are only a countable number of values of k for which the differential Equation (3) has a real solution $\psi(x)$ that is bounded for all x . For sufficiently large c , the eigenvalues have an asymptotic expansion (Flammer, 1957):

$$\begin{aligned}
k_n = & (2n+1) c - (2n^2 + 2n + 3) 2^{-2} - (2n+1)(n^2 + n + 3) 2^{-4} c^{-1} \\
& - 5(n^4 + 2n^3 + 7n + 3) 2^{-6} c^{-2} \\
& - (66n^5 + 165n^4 + 962n^3 + 1,278n^2 + 1,321n + 453) 2^{-10} c^{-3} \\
& - (252n^6 + 756n^5 + 5,885n^4 + 10,510n^3 + 18,478n^2 + 13,349n \\
& + 4,425) 2^{-12} c^{-4} \\
& - \left[527(2n+1)^7 + 61,529(2n+1)^5 + 1,043,961(2n+1)^3 \right. \\
& \left. + 2,241,599(2n+1) \right] 2^{-20} c^{-5} - \dots
\end{aligned}$$

For $c > n\pi$, the expansion gives a good approximation, the accuracy being better for smaller n . The problem of two or more nearly equal eigenvalues does not occur for the differential operator as it does for the integral operator. An exact expression for the determination of the eigenvalues of L_x is developed later in the discussion.

2.2 SOLUTION TO THE DIFFERENTIAL EQUATION

Two approaches have been attempted in obtaining the solutions to the differential equation

$$\frac{d}{dx} \left[(1 - x^2) \frac{d\psi_n}{dx} \right] + (k_n - c^2 x^2) \psi_n = 0 \quad (5)$$

- (i) Power series expansion about various points on $[-1, 1]$.
- (ii) Expansion in terms of Legendre polynomials.

2.2.1 Power Series Expansion

From the differential Equation (5), it can be seen that the linearly independent solutions consist of an even function and an odd function. The one that is bounded for all x is the desired solution to the present problem since ψ_n has been required to be an absolutely square integrable function on $[-1, 1]$. For ψ_n to fulfill this requirement implies that it is bounded through

$$\lambda_n \psi_n(u) = \int_{-1}^1 e^{jcux} \psi_n(x) dx$$

Since ± 1 are regular, singular points of the differential equation, a series solution expanded about any point in $(-1, 1)$ will have a convergence problem from the computational point of view for values of x close to ± 1 . However, for expansion about 1, which is a regular, singular point, the series will converge for all x in $(-1, 1]$. A similar behavior is exhibited for expansion about -1. In view of the even or odd property of the eigenfunction, expansion about 1 or -1 seems to be most convenient because the region of interest $[0, 1]$ is considerably smaller than the region of convergence of the series.

After the indicial equation is examined, the series solution of the form

$$\psi(x) = \sum_p A_p (1-x)^p$$

is taken. The recurrence formulas for the coefficients then are

$$A_1 = \frac{1}{2} (c^2 - k) A_0$$

$$A_2 = \frac{1}{8} [(c^2 - k + 2) A_1 - 2c^2 A_0]$$

$$A_p = \frac{1}{2p^2} [(p^2 - p + c^2 - k) A_{p-1} - 2c^2 A_{p-2} + c^2 A_{p-3}] \quad p \geq 3$$

The crucial role of k in computation of the coefficients is apparent from these expressions. The error in the coefficient A_p due to inaccuracy in k grows with p especially when k is comparable to c^2 . The value of $|A_p|$ increases with p until $p \cong c$ and then decreases asymptotically as $|A_p| \sim \frac{1}{2} |A_{p-1}|$ for large p . The number of terms in the series that are contributing significantly to the sum becomes larger as x gets farther away from 1. Since values of A_p are less accurately determined for large p , the series representation becomes poor for x close to 0. If the accuracy of the representation is not satisfactory in the neighborhood of $x = 0$, which can be determined by a comparison of the computed values of $\psi(x)$ and $\psi(-x)$ near $x = 0$, an improvement may be achieved by the expansion of another series solution about 0 with the solutions joined together.

2.2.2 Expansion in Series of Legendre Polynomials*

The differential Equation (5) can be compared with the differential equation satisfied by the Legendre polynomials P_n :

$$\frac{d}{dx} \left[(1 - x^2) \frac{dP_n}{dx} \right] + n(n+1) P_n = 0 \quad (6)$$

This comparison suggests a series of the form

$$\sum_0^{\infty} A_n P_n(x)$$

for the solution to Equation (5). From the recurrence relation

$$\begin{aligned} x^2 P_n(x) &= \frac{n(n-1)}{4n^2-1} P_{n-2}(x) + \left[\frac{n^2}{4n^2-1} + \frac{(n+1)^2}{(2n+1)(2n+3)} \right] P_n(x) \\ &+ \frac{(n+1)(n+2)}{(2n+1)(2n+3)} P_{n+2}(x) \end{aligned}$$

*Flammer, 1957; Stratton, et al., 1956.

and the substitution of the expression

$$\sum_0^{\infty} A_n P_n(x)$$

into Equation (5), there results

$$\begin{aligned} A_{n+2} c^2 \frac{(n+2)(n+1)}{(2n+3)(2n+5)} + A_n \left[n(n+1) - k + c^2 \frac{2n(n+1) - 1}{(2n-1)(2n+3)} \right] \\ + c^2 \frac{n(n-1)}{(2n-3)(2n-1)} A_{n-2} = 0 \end{aligned} \quad (7)$$

and $A_{-1} = A_{-2} = 0$. This expression is a difference equation of second order, and it admits two linearly independent solutions. As n becomes large, the ratio A_n/A_{n-2} of one solution behaves as $-4n^2/c^2$, and the other decreases to zero as $-c^2/4n^2$. The boundedness requirement on ψ implies that the latter should be chosen. Let

$$S(n, k) = \left[n(n+1) - k + c^2 \frac{2n(n+1) - 1}{(2n-1)(2n+3)} \right] \frac{(2n+3)(2n+5)}{(n+2)(n+1)c^2}$$

$$T(n) = \frac{n(n-1)(2n+3)(2n+5)}{(2n-3)(2n-1)(n+2)(n+1)}$$

The solution to the difference equation can be written as an infinite continued fraction:

$$\frac{A_{n+2}}{A_n} = - \frac{T(n+2)}{S(n+2, k) - \frac{T(n+4)}{S(n+4, k) - \frac{T(n+6)}{S(n+6, k) - \dots}}} \quad (8)$$

But $A_2/A_0 = -S(0, k)$; thus, the condition must exist that

$$S(0, k) = \frac{T(2)}{S(2, k) - \frac{T(4)}{S(4, k) - \frac{T(6)}{S(6, k) - \dots}}} \quad (9)$$

which determines a set of special values for k , the eigenvalues of the differential operator L_x .

Unlike the case of the coefficient for the power series in which the computation started with $n = 0$ and proceeded outward to larger n , the present case starts with large n and comes inward to 0. In view of the resemblance of Equations (5) and (6), it can be seen that the maximum $|A_n|$, which is the largest magnitude of the projection of ψ_p on P_n , should be in the neighborhood of $n = \sqrt{k}$. Since the higher-index coefficients are decaying very rapidly, the number of terms in the series needed to have five place accuracy in the computed eigenfunction for all $|x| \leq 1$ is very reasonable. Of the two representations of the eigenfunction, the latter seems to be more suitable from the computational viewpoint.

2.3 COMPUTATIONAL DETAIL

The steps in computation of the eigenfunctions and eigenvalues of the finite Fourier transform are given below.

(i) The asymptotic expansion for the eigenvalues of L_x was used to provide the first order approximation, and the "exact" (computed to nine places) values of k were computed by Equation (9) for the first N eigenvalues, where $N =$ twice the number of wavelengths in the aperture. For higher-index eigenvalues, the asymptotic expansion becomes poor and, hence, useless. As the index gets higher, even the step by step hunting (evaluate the expression

$$F(k) = S(0, k) - \frac{T(2)}{S(2, k) - \frac{T(4)}{S(4, k) - \frac{T(6)}{S(6, k) - \dots}}}$$

at a discrete set of points) becomes difficult. The step size must be taken smaller and smaller as the index increases. A typical curve for $F(k)$ appears roughly as shown in Figure 1; the Δ becomes smaller as the index increases. As will be demonstrated later, the eigenelements that have indices higher than $N+1$ play only a minor role in pattern synthesis. Thus, for all practical purposes, only the first $N+1$ eigenelements need to be used.

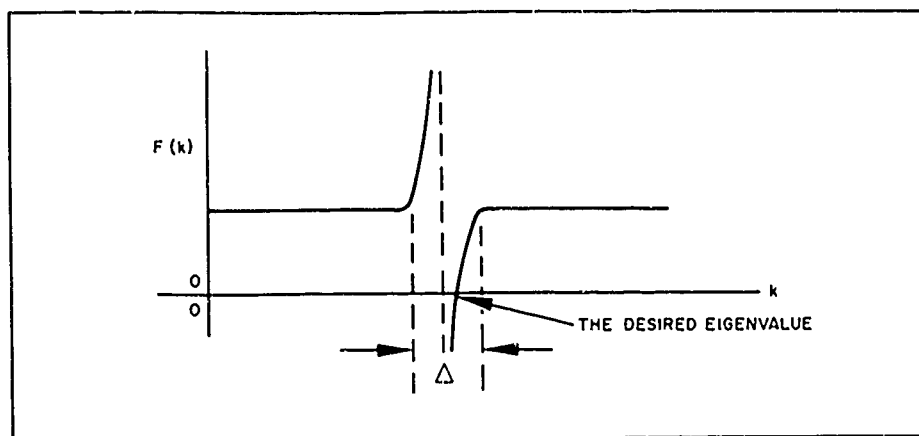


Figure 1. Typical curve for $F(k)$.

(ii) Two even (or odd) consecutive coefficients are computed from Equation (8) as the initial point in the recurrence process in Equation (7). For the range of c of interest, the number of significant terms in the summation is less than 150, so that the ratio A_{152}/A_{150} (or A_{151}/A_{149}) appears to be a suitable place to start. A_{150} (or A_{149}) was taken to be 10^{-40} so that the lower-index coefficient would not be too large and cause overflow on the machine. For $c = 10\pi$, the number of terms used in the summation $\sum A_n P_n(x)$ was 15 for the low index eigenfunctions and 30 for the higher ones. For $c = 25\pi$, the number of significant terms ranged from 25 to 55.

(iii) The eigenfunctions are computed with the values of k determined in step (i) and the two even (or odd) consecutive coefficients obtained in step (ii) as input data in a new computer program.

(iv) As a by-product from this program, the eigenvalues of the finite Fourier transform are obtained through the relationship

$$\lambda_n = \frac{1}{\psi_n(u)} \int_{-1}^1 \left\{ \begin{array}{l} \cos cux \\ j \sin cux \end{array} \right\} \psi_n(x) dx \quad n = \begin{array}{l} 0, 2, 4, \dots \\ 1, 3, 5, \dots \end{array} \quad (10)$$

by selecting a specific value for u on the right-hand side (of course, not such a value that $\psi_n(u) = 0$).

As a check on the program and the accuracy in determination of the parameters k and A_{152} (or A_{151}) from earlier computer programs for each n , hence each eigenfunction, various values of u were chosen in computing the eigenvalue from Equation (10). Since the left-hand side is independent of u , the results from various selections of u should be invariant. The first N eigenvalues determined by Equation (10) are accurate at least in the first four places after the decimal point. The increment in x was 0.005 in the integration. For higher-index eigenvalues, the accuracy in the result is believed to be within ± 5 to ± 10 percent, being less certain for higher index. The reason for the increasing uncertainty lies in the very rapid rise in value near 1 of the eigenfunctions; the slope becomes steeper as the index increases. The main contribution to the value of the integral, or the eigenvalue, is from the region near 1. The increment in x in $[0.9, 1]$ was taken as 0.0002 for the computation.

The graphs of some eigenfunctions for $c = 10\pi$ are shown in Figure 2. To show the variations in the eigenfunctions as c takes on various values, ψ_0 for $c = 10\pi, 15\pi, 20\pi, 25\pi$ is shown in Figure 3. A graph of the absolute value of the eigenvalues versus n for $c = 10\pi$ is shown in Figure 4. The absolute value of the eigenvalues remains essentially constant for the first $N-4$ eigenvalues and then decreases very rapidly after $n = N$. The eigenfunctions are tabulated in Section 3 for the argument in $[0, 1]$ in 0.01 increments. All computations were performed on the GE 265 machine using BASIC. The essential computer programs are given in the Appendix.

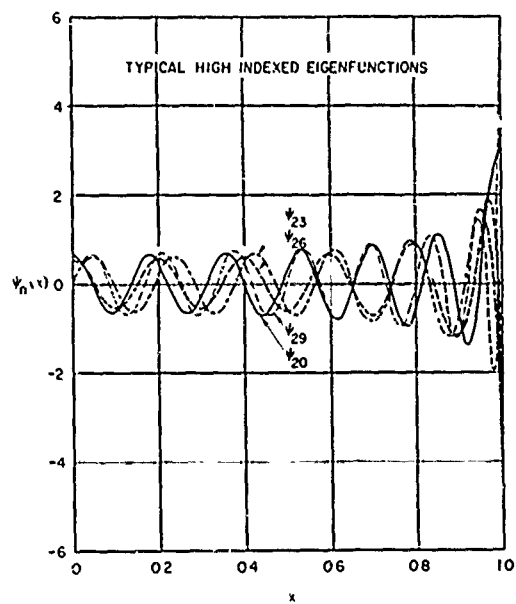
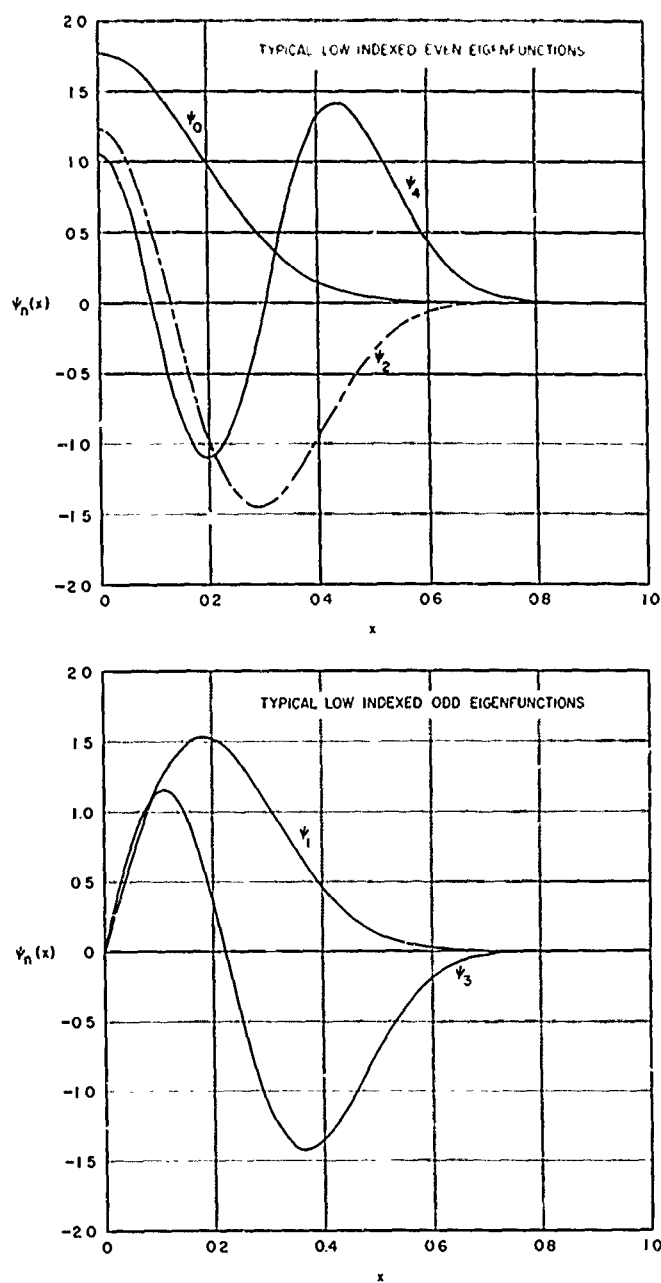


Figure 2. Typical eigenfunctions for $c = 10\pi$ and various ψ .

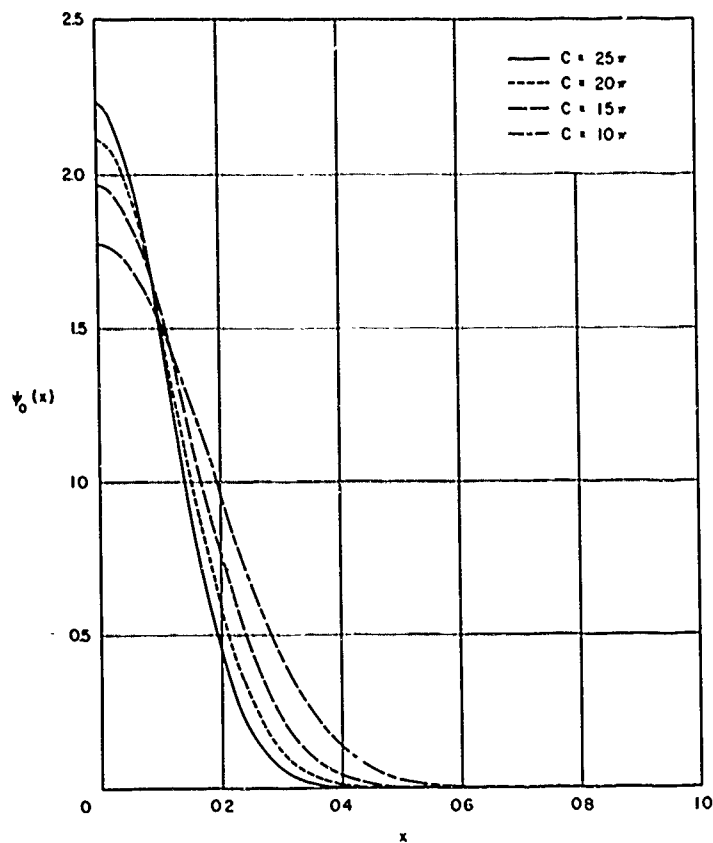


Figure 3. Effect on eigenfunctions of variations in c .

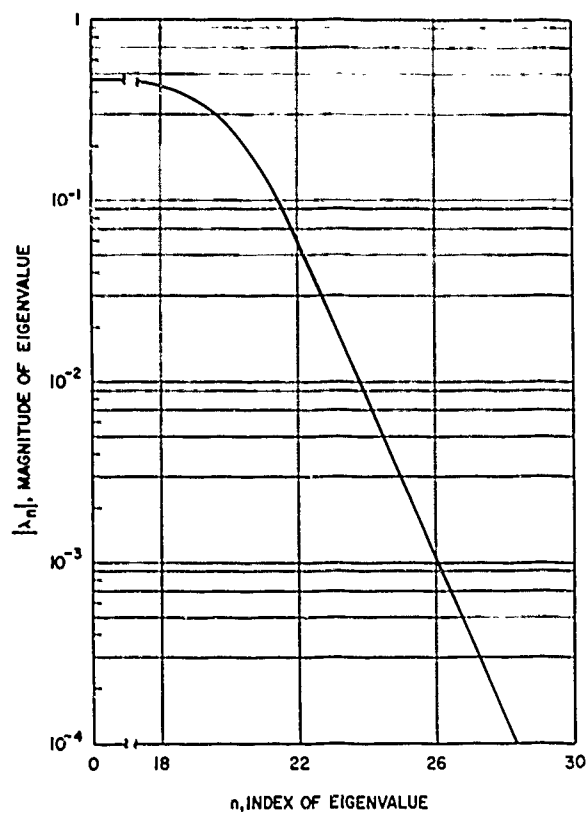


Figure 4. Magnitude of eigenvalues for $c = 10\pi$.

2.4 EXAMPLES OF PATTERN SYNTHESIS

An application of the synthesis technique that utilized the eigen-elements of the finite Fourier transform was formulated. An antenna was considered that had a radiation pattern

$$g(u) = \begin{cases} 1 - u & 0 \leq u \leq \frac{1}{2} \\ 0 & \text{elsewhere} \end{cases}$$

and an aperture length equal to ten wavelengths. First the generalized Fourier coefficients for g will be obtained via the relation

$$a_n = \int_{-1}^1 g(u) \psi_n(u) du \quad n = 0, 1, \dots$$

For a fixed superdirective ratio, γ , the Lagrange multiplier μ is computed from (Fong, 1965)

$$\sum_{n=0}^{29} \frac{a_n^2 \left(\frac{2\pi}{c|\lambda_n|^2} - \gamma \right)}{1 + \mu \left(\frac{2\pi}{c|\lambda_n|^2} - \gamma \right)^2} = 0$$

for $\mu \in \left[0, \frac{1}{\gamma - \frac{2\pi}{c\lambda_0^2}} \right]$. The summation goes to 29 only since there are

only 30 eigenfunctions available. With the value of μ obtained above, the coefficients \hat{a}_n , which provide the best approximation to g subject to the superdirective ratio constraint, are computed by

$$\hat{a}_n = \frac{a_n}{1 + \mu \left(\frac{2\pi}{|\lambda_n|^2} - \gamma \right)} \quad n = 0, 1, \dots, 29$$

The optimum aperture distribution is given by

$$\hat{f}(x) = \sum \frac{\hat{a}_n}{\lambda_n} \psi_n(x)$$

while the optimum approximating pattern is

$$\hat{g}(u) = \sum \hat{a}_n \psi_n(u)$$

The graphs of the approximating patterns (corresponding to different superdirective ratios of 1.2 and 10) and the aperture distributions are shown in Figure 5 and Figure 6, respectively.

The values of a_n , \hat{a}_n , and $a_n - \hat{a}_n$ corresponding to $\gamma = 1.2$ and 10 are listed in Table 2-1.

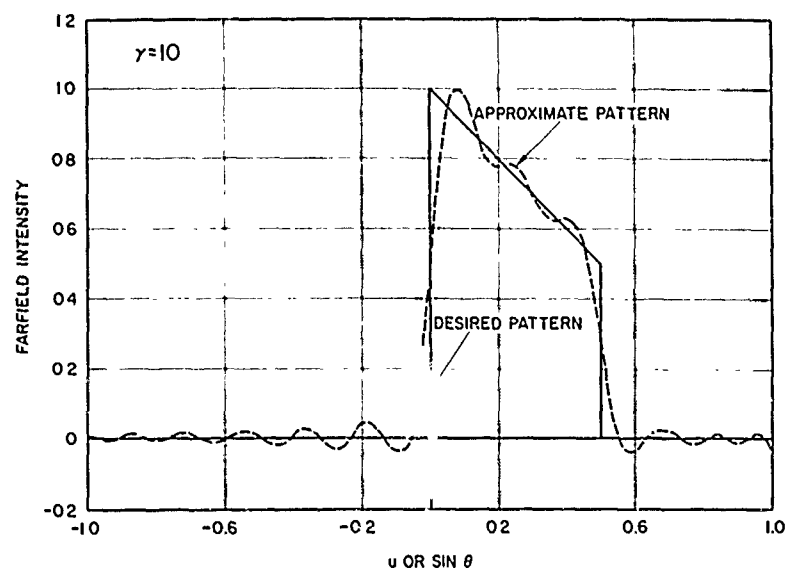
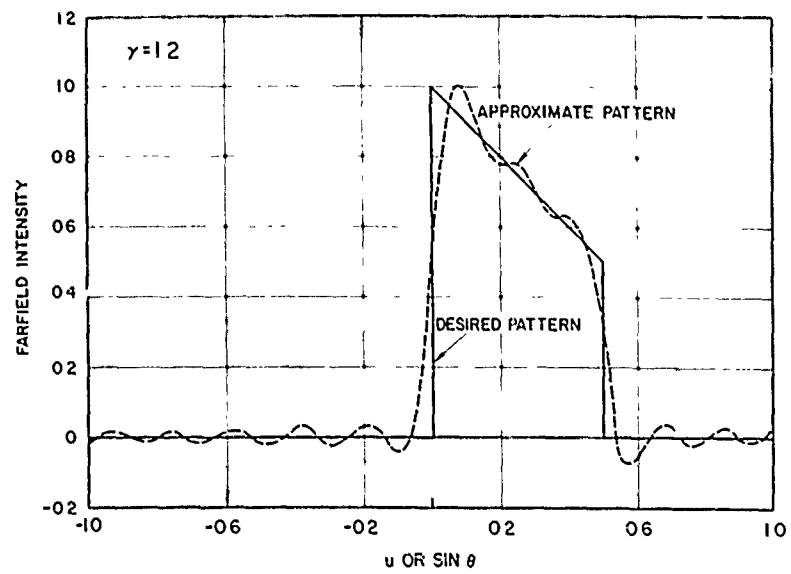


Figure 5. Pattern approximation by eigenelements for $c = 10\pi$.

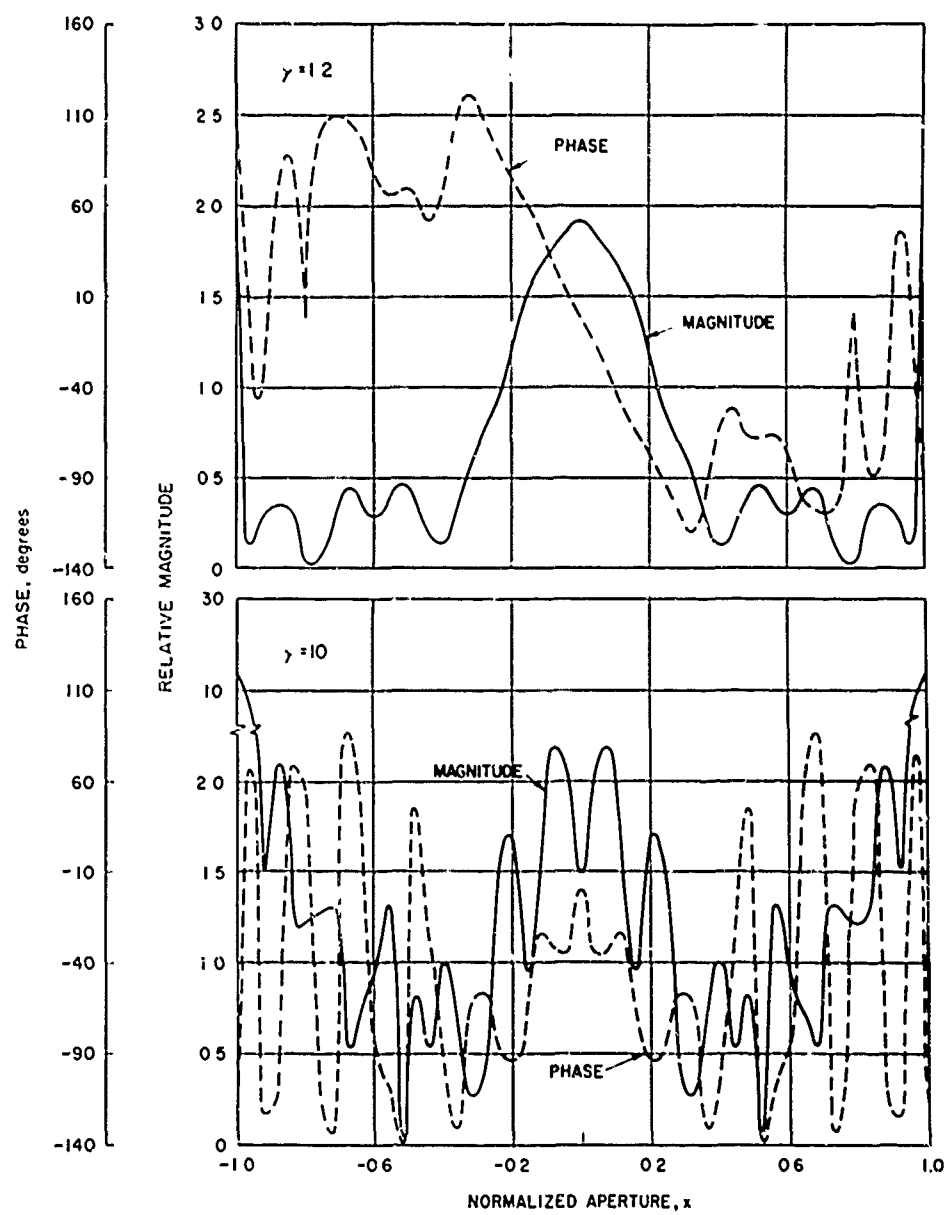


Figure 6. Aperture distributions
for $c = 10\pi$.

n	a_n	$\gamma = 1.2$		$\gamma = 10$	
		\hat{a}_n	$(a_n - \hat{a}_n) \times 10^5$	\hat{a}_n	$(a_n - \hat{a}_n) \times 10^5$
0	0.34425	0.34446	-21	0.34425	*
1	0.34367	0.34388	-21	0.34367	*
2	-0.14729	-0.14738	9	-0.14729	*
3	-0.03768	-0.03770	2	-0.03768	*
4	0.07564	0.07569	-5	0.07564	*
5	0.08009	0.08014	-5	0.08009	*
6	0.01558	0.01559	-1	0.01558	*
7	0.07356	0.07363	-7	0.07356	*
8	-0.02376	-0.02377	1	-0.02376	*
9	0.02850	0.02852	-2	0.02850	*
10	-0.01101	-0.01102	1	-0.01101	*
11	0.01044	0.01045	-1	0.01044	*
12	0.00995	0.00996	-1	0.00995	*
13	0.01588	0.01589	-1	0.01588	*
14	0.01760	0.01761	-1	0.01760	*
15	0.02581	0.02583	-2	0.02581	*
16	0.01353	0.01354	-1	0.01353	*
17	0.02982	0.02984	-2	0.02982	*
18	0.00594	0.00594	0	0.00594	*
19	0.02764	0.02761	3	0.02764	*
20	0.00024	0.00024	0	0.00024	*
21	0.02682	0.02607	75	0.02681	1
22	-0.00488	-0.00418	-70	-0.00487	-1
23	0.02373	0.01209	1164	0.02348	25
24	-0.00816	-0.00085	-731	-0.00746	-70
25	0.01672	0.00022	1650	0.00919	753
26	-0.00646	-0.00001	-645	-0.00089	-557
27	0.01023	*	1023	0.00021	1002
28	0.00097	*	97	*	97
29	0.00848	*	848	*	848

* Absolute value smaller than 10^{-6} .

Table 2-1. Generalized Fourier coefficients and eigenvalues.

It is interesting to note that there is very little improvement in the approximating radiation patterns when the superdirective ratio increases by greater than 8 times. This fact is apparent from an examination of Table 2-1 in which it can be seen that the significant contribution to the sum is from the first N terms. The high-index coefficients are essentially ignored because of the constraint. As might have been expected, when γ is near 1, greater emphasis is placed on the lower-index coefficients. While the improvement is small in changing the value of γ , the aperture distribution has altered tremendously. The rapid change in phase when $\gamma = 10$ contributes to high reactive power and, hence, the aperture is a high Q aperture in addition to being more difficult to construct.

Another approximating pattern was computed for an aperture of 20 wavelengths with $\gamma = 1.02$, and it is shown in Figure 7. As expected, the approximation improves considerably over that of the earlier cases.

Woodward's method was applied to the same problem; the pattern obtained is shown in Figure 8 for comparison. γ for this case is approximately 1.0003. It should be noted that the value of γ does not enter into this synthesis technique. For this particular value, the pattern obtained by using the eigenelements is essentially the same as either of the ones shown in Figure 5.

A summary of errors in the various approximations, in the mean square sense, is shown below. The error of approximating g by \hat{g} is given by

$$\begin{aligned} \|g - \hat{g}\| &= \left\{ \int_{-1}^1 |g(u) - \hat{g}(u)|^2 du \right\}^{1/2} \\ &= \left\{ \sum_0^{\infty} |a_n - \hat{a}_n|^2 \right\}^{1/2} \end{aligned}$$

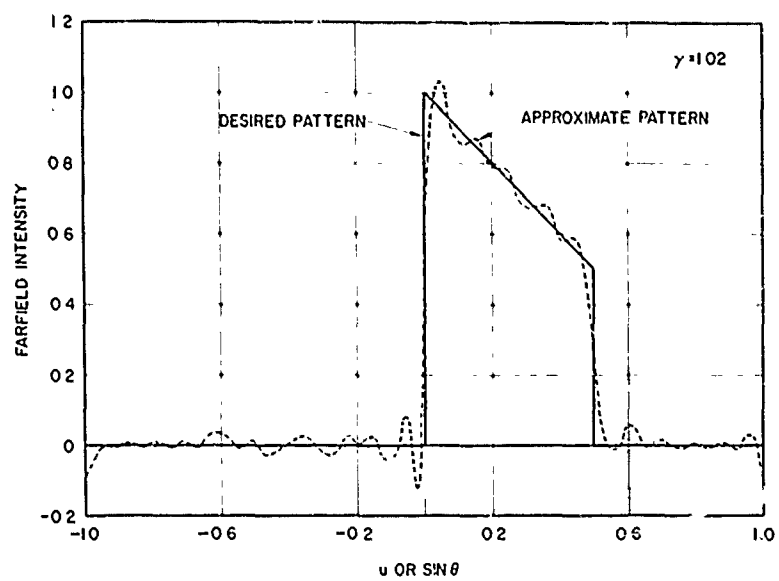


Figure 7. Pattern approximation by eigenelements for $c = 20\pi$.

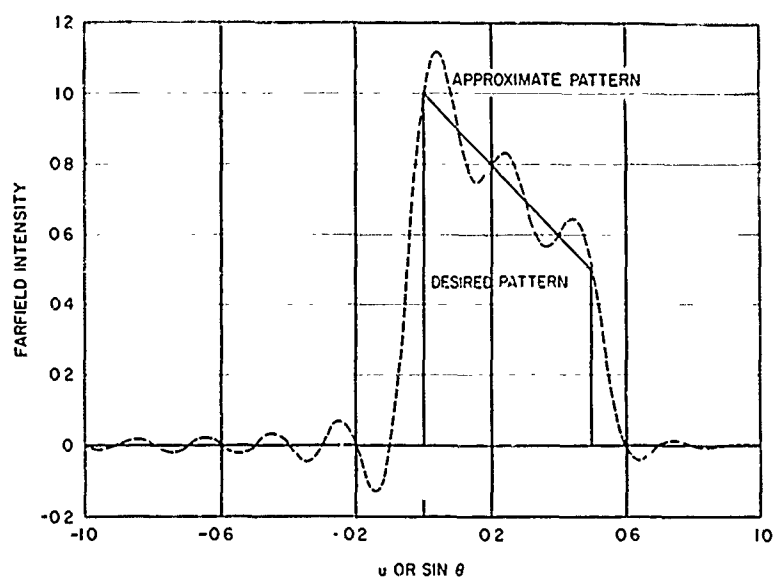


Figure 8. Pattern approximation by Woodward's method for $c = 10\pi$.

					Woodward
	$\gamma = 1.2$	$\gamma = 10$	$\gamma = 1.02$	$\gamma = 1.0003^*$	$\gamma = 1.0003$
	10 λ aperture	10 λ aperture	20 λ aperture	10 λ aperture	10 λ aperture
Error	0.0261	0.0162	0.0108	0.1005	0.2004

The norm of g is

$$\begin{aligned}
 \|g\| &= \left\{ \int_{-1}^1 |g(u)|^2 du \right\}^{1/2} \\
 &= \left\{ \int_0^{1/2} (1-u)^2 du \right\}^{1/2} \\
 &= 0.5401
 \end{aligned}$$

*This error is estimated; the actual error, being the optimum, is less than this figure, since the estimation was made by using the proper subspace spanned by the first 19 eigenfunctions. γ associated with elements in this subspace is less than 1.0003.

3.0 TABLES OF EIGENELEMENTS

The tables of eigenvalues and eigenfunctions for $c = 10\pi$, 15π , 20π , and 25π are presented in this section.

Table 3-1. Eigenvalues for $c=10\pi$ or aperture length = 10λ

n	λ_n	n	λ_n
0	.44721	15	-j.44710
1	j.44721	16	.44650
2	-.44721	17	j.44306
3	-j.44721	18	-.42565
4	.44721	19	-j.36716
5	j.44721	20	25497
6	-.44721	21	j.13733
7	-j.44721	22	-.05993
8	.44721	23	-j.02524
9	j.44721	24	.00848
10	-.44720	25	j.00288
11	-j.44720	26	-.00104
12	.44720	27	-j.00038
13	j.44720	28	.00014
14	-.44719	29	j.00005

Table 3-2. Eigenfunctions for $c = 10\pi$ or aperture length = 10λ

$$\psi_n(x) = F_n(x) \times 10^{-p}$$

x	$F_0(x)$	p	$F_1(x)$	p	$F_2(x)$	p	$F_3(x)$	p	$F_4(x)$	p	$F_5(x)$	p	$F_6(x)$	p	$F_7(x)$	p	x
0	1.77283		0		1.23759		0		1.05663		0		.949254		0		0
.01	1.77011		.138582		1.22812		.165703		1.04232		.180512		.931026		.189502		.01
.02	1.76199		.275713		1.19986		.327919		.939771		.355138		.877039		.370727		.02
.03	1.74853		.41076		1.15327		.483239		.830142		.51819		.789361		.535765		.03
.04	1.72985		.541926		1.08908		.628413		.655524		.664364		.671348		.677414		.04
.05	1.70613		.668268		1.00831		.760421		.717885		.788921		.727509		.789503		.05
.06	1.67756		.788712		.912234		.876539		.580998		.887836		.633334		.867155		.06
.07	1.64439		.902267		.802344		.774401		.428343		.957934		.183075		.907002		.07
.08	1.6069		1.00804		.680339		1.05205		.264008		.99693		.4.95587	4	.907323		.08
.09	1.56542		1.10529		.54808		1.10796		.9.23682	2	1.00379		.18655		.868121		.09
.1	1.52029		1.19322		.407555		1.14111		.6.20381	2	.978179		.565472		.791107		.1
.11	1.47187		1.27141		.260871		1.15094		.5.24636		.921035		.551131		.679624		.11
.12	1.42055		1.33942		.110065		1.13738		.4.20939		.834249		.677132		.53849		.12
.13	1.36673		1.39626		.4.26355	2	1.10088		.3.76673		.720642		.798046		.37378		.13
.14	1.31082		1.44387		.195154		1.0423		.2.71788		.581867		.889414		.122558		.14
.15	1.25324		1.48015		.3.545		.962981		.841059		.428278		.947904		.2.56013	3	.15
.16	1.19439		1.50589		.491581		.864626		.943174		.258777		.97142		.188136		.16
.17	1.13469		1.52133		.631742		.74929		1.0218		8.06541	2	.959173		.371466		.17
.18	1.07453		1.52679		.764283		.619314		.1.07515		.1.00524		.91169		.539719		.18
.19	1.01429		1.52271		.88774		.477262		.1.10208		.27944		.830776		.665862		.19
.2	.95433		1.50963		.1.00085		.325858		.1.10216		.450501		.715434		.803832		.2
.21	.894998		1.48815		.1.10257		.167922		.1.07558		.608703		.581729		.888778		.21
.22	.836609		1.45895		.1.19207		.6.30366	3	.1.02323		.749436		.422629		.93725		.22
.23	.779455		1.42274		.1.26877		.1.56184		.946569		.868684		.247802		.947322		.23
.24	.723797		1.3803		.1.33229		.3.1682		.847595		.963132		.6.34048	2	.918656		.24
.25	.66987		1.33241		.1.3825		.473034		.728788		.1.03025		.124155		.852487		.25
.26	.617874		1.27987		.1.41945		.622455		.593011		.06835		.308441		.751549		.26
.27	.56798		1.2235		.1.44342		.762947		.448427		.1.07659		.483232		.61994		.27
.28	.52033		1.16408		.1.45484		.892651		.2834		.1.05502		.642721		.462926		.28
.29	.475032		1.10238		.1.45434		.1.01		.1.16408		.1.00449		.781713		.286717		.29
.3	.432167		1.03914		.1.44266		.1.11376		.5.40533	2	.926622		.895786		.9.81906	2	.3
.31	.391789		.975068		.1.42066		.1.20298		.1.20298		.823755		.98142		9.53393	2	.31
.32	.353921		.910803		.1.3893		.1.27708		.391896		.698804		.1.03609		.286688		.32
.33	.318567		.846948		.1.34962		.1.35576		.552966		.555177		.1.05832		.468488		.33
.34	.285706		.784046		.1.30269		.1.37904		.705047		.396645		.1.04768		.634161		.34
.35	.255294		.722576		.1.24961		.1.4072		.845737		.227218		.1.00479		.777712		.35
.36	.227274		.66296		.1.19148		.1.42078		.973016		.051017		.931233		.894082		.36
.37	.20157		.605553		.1.12938		.1.42056		1.08527		.127847		.829473		.979296		.37
.38	.178094		.550652		.1.06436		.1.40748		1.18129		.303392		.702738		1.03057		.38
.39	.156747		.498492		.997417		.1.38266		1.26032		.477868		.554876		1.04637		.39
.4	.13742		.449249		.929476		.1.34732		1.35198		.641849		.390206		1.02643		.4
.41	.12		.403048		.861397		.1.30278		1.36629		.794307		.213352		.971721		.41
.42	.104368		.359962		.793954		.1.2504		1.39364		.832671		2.90819	2	.884331		.42
.43	.9.0435	2	.320016		.727833		.1.19155		1.40471		1.05487		.157848		.767374		.43
.44	.7.9836	2	.283197		.663628		.1.1276		1.4005		1.15935		.342846		.624819		.44
.45	.6.9877	2	.249453		.601841		.1.05287		1.3822		1.24509		.321613		.461309		.45
.46	.5.72967	2	.218702		.542884		.989607		1.35121		1.31156		.690261		.281965		.46
.47	.4.67932	2	.190637		.487078		.917995		1.30905		1.35874		.842403		.9.21825	2	.47
.48	.4.13716	2	.163725		.434663		.846167		1.25734		1.38706		.984226		.1.102572		.48
.49	.3.49198	2	.143222		.385795		.774903		1.19771		1.39733		.1.10453		.29695		.49
.5	.2.93392	2	.123168		.340574		.709231		1.15183		1.39073		.1.20475		.4859		.5
.51	.2.45354	2	.105395		.299011		.637813		1.06128		1.3687		.1.28397		.664816		.51
.52	.2.04204	2	.8.97308	2	.261079		.573246		.987599		1.3329		.1.34186		.829663		.52
.53	.0.91913	2	.7.60031	2	.226694		.512011		.912212		1.28517		.1.3787		.977076		.53
.54	.1.39385	2	.6.40397	2	.195735		.454469		.83642		1.2274		.1.39524		.1.10442		.54
.55	.1.14288	2	.5.36728	2	.1.68045		.400874		.761384		1.16154		.1.39272		.1.20982		.55
.56	.9.32239	3	.4.47406	2	.143443		.35138		.688113		1.08952		.1.37272		.1.29216		.56
.57	.7.56378	3	.037089		.121728		.306049		.61746		1.01318		.1.33713		.1.35108		.57
.58	.6.10350	3	.3.05727	2	.102688		.264866		.55012		.934267		.1.28802		.1.3869		.58
.59	.4.89765	3	.2.50361	2	.8.61042	2	.227747		.486638		.854368		.1.22762		.1.40054		.59
.6	.3.90751	3	.2.04138	2	.7.17549	2	.194553		.42741		.774916		.1.15817		.1.39347		.6
.61	.3.09919	3	1.65313	2	.5.94222	2	.165097		.372698		.697156		.1.08191		.1.3676		.61
.62	.2.44320	3	1.33043	2	.4.88943	2	.139158		.32264		.622139		.1.00101		.1.32515		.62
.63	.1.91405	3	1.06392	2	.3.99683	2	.1.16493		.277265		.550722		.917461		.1.26857		.63
.64	.1.48988	3	.8.45238	3	.3.24528	2	.7.68394	2	.236511		.483568		.833118		.1.20046		.64
.65	.1.13203	3	.6.66994	3	.2.61693	2	.7.99285	2	.200234		.421159		.749607		.1.12342		.65
.66	.8.48920	4	.5.22694	3	.2.09534	2	.6.54905	2	.1.68229		.363804		.668332		.1.04		.66
.67	.6.74584	4	.4.06686	3	.1.65533	2	.5.32603	2	1.40241		.311664		.590456		.932634		.67
.68	.5.10607	4	.3.14090	3	.1.31398	2	.4.29825	2	.1.15982		.264759		.516905		.865359		.68
.69	.3.85553	4	.2.40724	3	.1.02863	2	.034415	2	.9.51411	2	.222999		.448371		.774774		.69
.7	.2.85841	4	1.83035	3	.7.98816	3	.2.73318	2	.7.73954	2	.186196		.385325		.688013		.7
.71	.2.11273	4	1.38028	3	.6.15223	3	.2.15248	2	.6.02421	2	.154089		.328041		.604721		.71
.72	.1.54824	4	1.03199	3	.4.69764	3	.1.68049	2	.4.99002	2	.12636		.276612		.526052		.72
.73	.1.12445	4	.7.64721	4	.3.55501	3	.1.30023	2	.039528		.102655		.23098		.452871		.73
.74	.8.09023	5	.5.61406	4	.2.66534	3	.9.96641	3	.5.10169	2	.082595		.190957		.385772		.74
.75	.5.76398	5	.4.08143	4	.1.97896	3	.7.56928	3	.2.41007	2	.6.57955	2	.156257		.325099		.75
.76	.4.06451	5	.2.93699	4	.1.45446	3	.5.68452	3	1.85363	2	.051874		.126517		.270972		.76
.77	.2.83516	5	.2.09083	4	.1.05760	3	.4.22608	3	1.41054	2	.040461		.101324		.223322		.77
.78	.1.95501	5	1.47166	4	.7.60433	4	.3.10691	3	1.06147	2	.5.12075	2	.8.02324	2	.181923		.78
.79	1.33188	5	1.02352	4	.5.40319	4	.2.25741	3	.7.89475	3	2.37899	3	.062785		.1.64225		.79
.8	.8.35751	6	.7.02859	5	.3.79131	4	.1.61993	3	.5.79980	3	1.79135	2	.4.85286	2	.1.16388		.8
.81	.5.94126	6	.4.76166	5	.2.62506	4	.1.14727	3	.4.20553	3	1.33148	2	.3.73261	2	.9.13115	2	.81
.82	.3.88293	6	.3.17975	5	.1.79192	4	.8.00759	4	.3.00759	3	.7.76182	3	.2.78665	2	.7.06627	2	.82
.83																	

TABLE 3-2. (continued)

x	F ₀ (x)	p	F ₀ (x)	p	F ₁₀ (x)	p	F ₁₁ (x)	p	F ₁₂ (x)	p	F ₁₃ (x)	p	F ₁₄ (x)	p	F ₁₅ (x)	p	x
0	.871835		0		.809538		0		.755149		0		.703694		0		0
.01	.870381		.147418		.785461		.176955		.725038		.196384		.676198		.19242		.01
.02	.868926		.318879		.761467		.381267		.652505		.378374		.55985		.369165		.02
.03	.867471		.492497		.601317		.5411		.530816		.532438		.4689		.515506		.03
.04	.865916		.674705		.45215		.666191		.372349		.647382		.305228		.612476		.04
.05	.864361		.857427		.275985		.748501		.188012		.714685		.117571		.672509		.05
.06	.862806		.104198		.832446	2	.782732		.504072	3	.729372		.74609	2	.670212		.06
.07	.861251		.2840342		.6114673		.766663		.206605		.690322		.270523		.612722		.07
.08	.859696		.4680339		.4306067		.701294		.389715		.600359		.440727		.504703		.08
.09	.858141		.656644		.479627		.590773		.546223		.466036		.576792		.354963		.09
.1	.856586		.845178		.6251		.442134		.665337		.297163		.668082		.175737		.1
.11	.855031		.1037203		.733892		.64841		.738831		.106085		.70743		.183121	2	.11
.12	.853476		.280919		.799572		.701868	2	.761601		.932214	2	.691691		.211289		.12
.13	.851921		.4680747	2	.818245		.129436		.732016		.286166		.621994		.387364		.13
.14	.850366		.656339		.108069		.788775		.652026		.458613		.503651		.532067		.14
.15	.848811		.844744		.299985		.712842		.527029		.59791		.345753		.633472		.15
.16	.847256		.103746		.476447		.59484		.634279		.693812		.160464		.683167		.16
.17	.845701		.282807		.62807		.44161		.735424		.739225		.379142	2	.676953		.17
.18	.844146		.468005		.746805		.262031		.79022		.730724		.234045		.619185		.18
.19	.842591		.656643		.65016	2	.795129		.220836		.668799		.41274		.502749		.19
.2	.841036	2	.842533		.133673		.749755		.405812		.55782		.560131		.348678		.2
.21	.839481		.103746		.26936		.656859		.5633		.405719		.664737		.165412	2	.21
.22	.837926		.280919		.502149		.322178		.684415		.223414	2	.718352		.321952		.22
.23	.836371		.4680747		.649271		.554059		.759027		.240231		.716672		.288089		.23
.24	.834816		.656339		.759714		.162926		.782612		.178086		.659634		.406309		.24
.25	.833261		.844744		.827258		.393723	2	.75344		.36833		.551423		.532282		.25
.26	.831706		.103746		.221255		.647957		.240305		.673319		.400151		.654009		.26
.27	.830151		.282807	2	.820568		.427437		.547466		.660036		.217248		.703036		.27
.28	.828596		.468005		.746559		.589125		.384147		.740306		.016583		.695146		.28
.29	.827041		.656643		.630007		.715576		.194126		.767837		.186581		.630708		.29
.3	.825486		.845178		.477352		.798752		.100612	2	.740454		.376753		.514658		.3
.31	.823931		.103746		.297016		.833555		.214937		.659883		.539387		.356108		.31
.32	.822376		.280919		.59236	2	.817685		.406977		.53163		.661986		.167628		.32
.33	.820821		.4680747		.106066		.752021		.573496		.564584		.735045		.376117	2	.33
.34	.819266		.656339		.30676		.640356		.703479		.170388		.752777		.23778		.34
.35	.817711		.844744		.492248		.489249		.788295		.373868	2	.713544		.42219		.35
.36	.816156	2	.103746		.652494		.307596		.822245		.244192		.619987		.574094		.36
.37	.814601		.282807		.72606		.778868		.802924		.455525		.478831		.681124		.37
.38	.813046		.468005		.618876		.864592		.731355		.597938		.300381		.734432		.38
.39	.811491		.656643		.505078		.308815		.611915		.71997		.97631	2	.729401		.39
.4	.809936		.845178		.267695		.898142		.45203		.792938		.114063		.666022		.4
.41	.808381		.103746	2	.64919		.844086		.26169		.811527		.319401		.348906		.41
.42	.806826		.280919		.139769		.745641		.527977	2	.77415		.502979		.386932		.42
.43	.805271		.4680747		.607786		.866749		.161591		.683041		.691079		.192559		.43
.44	.803716		.656339		.330761		.457456		.368096		.344024		.732943		.191413	2	.44
.45	.802161		.844744		.698333		.243149		.89965		.536847		.59585		.231659		.45
.46	.800606		.103746		.837392		.344669		.814519		.707277		.788361		.428332		.46
.47	.800051		.282807		.942132		.178389		.700509		.818823	2	.560744		.395638		.47
.48	.798496		.468005		.100831		.385136		.547219		.881496		.272906		.714407		.48
.49	.796941		.656643		.103337		.575923		.361666		.891272		.474213		.780843		.49
.5	.795386		.845178		.101692		.741791		.15411		.847311		.646601		.22879		.5
.51	.793831	2	.103746		.958642		.875086		.645061	2	.715962		.778567		.897502	3	.51
.52	.792276		.280919		.862246		.96978		.282765		.50592		.861235		.214549		.52
.53	.790721		.4680747		.731251		.102171		.489391		.431221		.888915		.425995		.53
.54	.789166		.656339		.570772		.10287		.673836		.224017		.85944		.610374		.54
.55	.787611		.844744		.386845		.990605		.826808		.675305	4	.774277		.754542	2	.55
.56	.786056		.103746		.186133		.909209		.940718		.22628		.638396		.848102		.56
.57	.784501		.282807	2	.43833		.788094		.101001		.444225		.459922		.884116		.57
.58	.782946		.468005		.327704		.632379		.103138		.641145		.249591		.849555		.58
.59	.781391		.656643		.447092		.448423		.100387		.80629	2	.200507	2	.77549		.59
.6	.779836		.845178		.243481		.928808		.930737		.214941		.636993		.844677		.6
.61	.778281		.103746	2	.53365		.809674		.100783		.441365		.492766		.879577		.61
.62	.776726		.280919		.198071		.651827		.103346		.645792		.234555	3	.850208		.62
.63	.775171		.4680747		.418992		.462168		.100622		.816169		.574878		.175786		.63
.64	.773616		.656339		.630208		.248732		.927394		.942494		.246846		.608319		.64
.65	.772061		.844744		.832774		.202484	2	.800753		.101736		.479227		.411483		.65
.66	.770506		.103746		.998769		.214304		.632293		.10363		.686072		.18061		.66
.67	.768951		.282807		.14643		.446135		.429817		.997986		.854198		.686074	2	.67
.68	.767396		.468005		.126527		.667032		.20245		.904159		.972855		.319151		.68
.69	.765841		.656643		.135354		.86971		.398869	2	.759468		.103435		.553857		.69
.7	.764286		.845178		.141079		.104807		.28702		.571083		.103444		.75658		.7
.71	.762731		.103746		.143769		.119741		.529047		.348196		.972527		.913278		.71
.72	.761176		.280919		.143595		.13145		.756815		.101435		.851555		.101292		.72
.73	.759621		.4680747		.140812		.139766		.962322		.15779		.677755		.104815		.73
.74	.758066		.656339		.135743		.144663		.113904		.417936		.460147		.101574		.74
.75	.756511		.844744		.128756		.146255		.128211		.667959		.209903		.916667		.75
.76	.754956		.103746		.120248		.144771		.138849		.897869		.603904	2	.75599		.76
.77	.753401		.282807		.110622		.140536		.145692		.10992		.337533		.542385		.77
.78	.751846		.468005		.10027		.133948		.148785		.126536		.6085		.287503		.78
.79	.750291		.656643		.895624		.125452		.148327		.139185		.861182		.512915	3	.79
.8	.748736		.845178		.788512		.115515		.144649		.147626		.108503		.289757		.8
.81	.747181		.103746		.683644		.104603		.138181		.151842		.127155		.581563		.81
.82	.745626		.280919		.583994		.931612		.129425		.152005		.141469		.897029		.82
.83	.744071		.4680747		.491203		.815967		.118924		.148456		.151095		.110209		.83
.84	.742516	2	.656339		.406608		.702653		.107276		.141674		.155946		.130651		.84
.85	.740961	2	.844744		.331074		.594631		.948653		.132236		.156181		.146272		.85
.86	.739406	2	.103746		.264793		.494216		.823321		.120775		.152172		.13659		.86
.87	.737851	2	.282807	2	.207923		.403										

TABLE 3-2. (continued)

x	F ₁₆ (x)	p	F ₁₇ (x)	p	F ₁₈ (x)	p	F ₁₉ (x)	p	F ₂₀ (x)	p	F ₂₁ (x)	p	F ₂₂ (x)	p	F ₂₃ (x)	p	x
0	.648677		0		.578864		0		.545372		0		.52833		0		0
.01	.520869		.18291		.551834		.168197		.518299		.179983		.556682		.202596		.01
.02	.559233		.349389		.473259		.32021		.459757		.341581		.475074		.383029		.02
.03	.411045		.484475		.350446		.441406		.317515		.468281		.333203		.521355		.03
.04	.247268		.576017		.194824		.520106		.163665		.547117		.15597		.602977		.04
.05	.620103	2	.615761		.208765	2	.548708		.526380	3	.570001		.379396	2	.565934		.05
.06	.128788		.600095		.155196		.524412		.176311		.53454		.227971		.565934		.06
.07	.308697		.53037		.316981		.4495		.528747		.444292		.394128		.451372		.07
.08	.46221		.412766		.449378		.3311		.448734		.308408		.518788		.287124		.08
.09	.576072		.257841		.54		.180602		.524322		.14071		.588705		.911011	2	.09
.1	.640419		.793977	2	.580331		.123855	2	.547933		.417107	2	.596389		.115278		.1
.11	.64963		.106556		.566512		.157367		.517139		.220225		.540897		.309415		.11
.12	.60281		.283332		.497741		.312311		.34861		.376564		.427355		.469995		.12
.13	.503874		.435033		.386105		.437494		.309148		.494675		.26317		.579318		.13
.14	.361221		.547979		.236033		.520784		.1536		.562371		.817396	2	.625246		.14
.15	.187015		.611932		.635971	2	.554048		.203287	2	.572577		.114983		.602543		.15
.16	.384798	3	.621017		.115381		.533934		.190955		.52407		.330163		.513478		.16
.17	.195008		.57426		.784163		.462213		.343446		.421623		.4541		.367601		.17
.18	.370043		.475684		.426992		.345621		.4663		.273338		.56035		.180731		.18
.19	.513871		.333961		.530467		.195227		.535592		.100617		.607469		.267469	2	.19
.2	.614043		.161652		.584803		.253909	2	.555664		.853277	2	.590216		.232094		.2
.21	.66181		.259087	2	.584748		.147606		.520866		.51014		.590216		.412697		.21
.22	.652884		.211968		.530084		.307113		.453891		.263282		.37545		.548546		.22
.23	.567809		.379849		.426669		.437705		.303284		.414963		.324688		.624445		.23
.24	.471929		.514436		.281003		.326667		.141634		.580991		.275026	3	.631709		.24
.25	.314946		.603527		.109365		.565225		.345702	2	.577814		.195981		.569155		.25
.26	.150103		.638934		.073399		.549406		.208473		.515159		.374887		.443286		.26
.27	.669283	2	.617222		.250337		.480441		.362572		.599121		.514815		.267625		.27
.28	.25938		.540037		.404951		.364673		.481428		.241301		.600629		.612798	2	.28
.29	.430803		.413977		.52273		.21298		.553008		.576591	2	.62284		.133117		.29
.3	.560468		.250027		.592502		.397531	2	.567892		.133091		.578654		.351875		.3
.31	.65462		.626021	2	.607484		.13846		.530042		.311379		.472317		.512851		.31
.32	.68748		.131718		.269532		.304526		.437033		.45877		.314724		.617898		.32
.33	.561926		.315648		.471334		.442368		.299744		.559864		.122314		.654915		.33
.34	.579765		.472729		.33211		.538508		.131513		.603896		.844625	2	.619233		.34
.35	.447606		.58879		.160871		.583376		.051148		.585876		.284091		.514207		.35
.36	.276316	2	.653212		.267233	2	.572255		.230163		.387671		.37518		.350925		.36
.37	.801244		.659886		.213389		.505767		.387671		.37518		.37518		.147061		.37
.38	.124559		.607781		.381802		.389854		.307793		.203512		.639955		.750136	2	.38
.39	.320517		.50106		.515196		.23525		.578227		.910049	3	.633785		.290659		.39
.4	.491161		.548743		.60382		.564931	2	.591492		.188049		.559313		.475682		.4
.41	.621925		.163944	2	.636143		.129451		.54572		.367587		.423846		.609032		.41
.42	.701494		.372597		.609665		.304696		.44489		.510766		.241309		.675183		.42
.43	.722758		.372221		.526276		.452529		.228497		.602384		.03084		.669944		.43
.44	.68543		.418262		.393132		.55784		.120663		.63239		.165192		.581469		.44
.45	.586255		.364226		.222037		.61063		.712075	2	.596973		.383544		.43036		.45
.46	.458809		.661898		.283987	2	.605044		.25812		.499038		.542574		.228828		.46
.47	.252894		.702185		.170148		.540958		.421352		.347985		.64459		.100924	3	.47
.48	.435867	2	.680937		.355339		.423851		.544307		.158831		.677802		.235314		.48
.49	.171989		.599348		.509245		.26434		.614201		.492645	2	.637681		.442359		.49
.5	.37606		.463888		.619357		.772517	2	.62335		.254915		.527559		.603473		.5
.51	.551655		.285787		.672344		.119847		.570027		.43668		.352392		.697804		.51
.52	.684		.800977	2	.665064		.308091		.586974		.57528		.147712		.713556		.52
.53	.751732		.135578		.595631		.469292		.239656		.655617		.821579	2	.647666		.53
.54	.777845		.342587		.47018		.587578		.10812		.668407		.306426		.506322		.54
.55	.730269		.52282		.29943		.650958		.972571	2	.611231		.500473		.304447		.55
.56	.62087		.660279		.983644	2	.652506		.296068		.48891		.64252		.06419		.56
.57	.46129		.742464		.115071		.591097		.48891		.313131		.71605		.187446		.57
.58	.260175		.761413		.32156		.47159		.596337		.101367		.711712		.215177		.58
.59	.543842	2	.714722		.502138		.304462		.666433		.124823		.628489		.6107		.59
.6	.198326		.605212		.639913		.1049		.670537		.34195		.473999		.731889		.6
.61	.419475		.441325		.721606		.108564		.606982		.526993		.263862	2	.769548		.61
.62	.611514		.236171		.738793		.315744		.480818		.6598		.201859		.717407		.62
.63	.758222		.652538	3	.668728		.496684		.30344		.725245		.230699		.575505		.63
.64	.847932		.22854		.574664		.633542		.915906	2	.714921		.461001		.730024		.64
.65	.872494		.44923		.405652		.712305		.134184		.628175		.644499		.111942		.65
.66	.6628911		.636668		.195821		.724183		.351485		.47237		.759465		.16537		.66
.67	.71938		.774467		.368009	2	.666528		.538233		.262327		.791176		.429378		.67
.68	.551141		.850108		.271815		.543211		.67486		.189817		.733852	2	.648095		.68
.69	.335942		.856002		.488215		.364385		.746303		.232632		.591394		.79587		.69
.7	.891696	2	.790161		.666219		.145676		.743604		.465876		.377466		.846828		.7
.71	.1713		.656409		.78902		.931557	2	.664961		.655244		.114222		.79753		.71
.72	.426411		.46413		.844295		.329961		.516093		.779067		.169836		.648533		.72
.73	.657324		.275267		.825353		.542196		.3099		.821918		.442795		.414608		.73
.74	.846777		.352698	2	.731804		.708996		.54613	2	.776437		.672779		.121517		.74
.75	.98031		.303991		.569705		.813138		.644413		.831528		.831528		.196564		.75
.76	.104727		.557401		.351156		.842718		.442383		.644413		.897746		.500775		.76
.77	.104151		.775136		.933836	2	.79237		.654336		.17363		.859844		.732059		.77
.78	.961797		.939219		.182618		.663941		.806879		.119076		.717732		.915951		.78
.79	.811763		.103544		.453765		.46652		.882089		.410387		.483443		.967107		.79
.8	.599614		.10544		.696754		.21583		.868962		.668186		.180437		.893046		.8
.81	.5337454		.992222		.889964		.69732	2	.764796		.862339		.158398		.696595		.81
.82	.403692	2	.850805		.101522		.357018		.57574		.967959		.493928		.396632		.82
.83	.274663		.637623		.105928		.627774		.316642		.968314		.784851		.270669	2	.83
.84	.589971		.365133		.101496		.853308		.100228	2	.057053		.952309		.366426		.84
.85	.886391		.049801		.881704		.101051		.315654		.63958		.108458		.731006		.85
.86	.15438		.289149		.665696		.108115		.628596		.333203		.104138		.101293		.86
.87	.137496		.631163		.379366		.105347		.895382		.336869</						

TABLE 3-2. (continued)

x	$F_{24}(x)$	p	$F_{25}(x)$	p	$F_{26}(x)$	p	$F_{27}(x)$	p	$F_{28}(x)$	p	$F_{29}(x)$	p	x
0	.736295		0		.657244		0		.672186		0		0
.01	.660385		.217649		.61773		.234185		.627507		.47408		.01
.02	.47644		.413386		.501237		.438488		.439392		.460632		.02
.03	.334779		.558542		.323305		.586825		.311536		.610192		.03
.04	.155727		.637384		.10553		.660225		.659385	2	.67937		.04
.05	.736295	2	.641177		.125333		.649252		.17506		.647085		.05
.06	.484215		.56305		.341128		.555215		.396648		.529135		.06
.07	.458524		.423567		.515158		.390014		.56567		.537716		.07
.08	.480254		.733094		.66604		.174653		.659558		.099207		.08
.09	.636077		.193356	2	.66303		.063423		.665673		.153445		.09
.1	.617517		.201741		.517731		.233802		.583012		.385253		.1
.11	.53195		.403338		.489347		.486946		.422373		.564015		.11
.12	.594173		.551722		.306426		.618103		.204966		.664733		.12
.13	.191857		.638041		.746783	2	.670193		.403701	2	.573298		.13
.14	2.28671		.648878		.131144		.636335		.280966		.592103		.14
.15	.235532		.582704		.36331		.520567		.484653		.420709		.15
.16	.422049		.4471		.534226		.337464		.62405		.194116		.16
.17	.560865		.257898		.637615		.110305		.680235		.603239	2	.17
.18	.636005		.573545	2	.666185		.131864		.645322		.307219		.18
.19	.638621		.186456		.610305		.357918		.573579		.51203		.19
.2	.568076		.392701		.478513		.538633		.330934		.659893		.2
.21	.43716		.550967		.706769		.650457		.929303	2	.683673		.21
.22	.246055		.644169		.85479	2	.678573		.158635		.636746		.22
.23	.308433	2	.660848		.17801		.618876		.387922		.433965		.23
.24	.188757		.598586		.593546		.478583		.569585		.80874		.24
.25	.388148		.464358		.561107		.275332		.673004		2.70536	2	.25
.26	.543731		.273774		.659513		.528862	2	.685669		.231937		.26
.27	.637546		.492869	2	.676027		.210753		.605232		.459506		.27
.28	.658335		.182426		.60832		.430717		.441944		.623172		.28
.29	.603376		.393611		.463383		.595714		.717338		.699204		.29
.3	.478231		.558743		.259673		.683754		.382741	2	.676074		.3
.31	.297057		.657605		.219699	2	.68269		.290402		.556242		.31
.32	.801722	2	.677757		.220129		.591912		.504443		.355375		.32
.33	.147546		.616118		.438185		.422562		.650695		.103216		.33
.34	.359828		.479424		.598724		.196219		.708418		.166255		.34
.35	.531917		.283541		.686721		.575777	2	.6888		.413823		.35
.36	.643466		.516549	4	.623793		.30618		.536331		.603544		.36
.37	.680934		.188435		.602467		.515962		.328379		.707348		.37
.38	.639235		.407616		.489156		.656875		.072989		.70922		.38
.39	.522428		.57897		.218392		.715158		.194828		.60771		.39
.4	.543368		.681067		.524078	2	.676309		.437839		.416396		.4
.41	.122323		.700642		.481405		.546399		.621725		.162176		.41
.42	.115277		.634339		.496691		.321737		.719925		.118411		.42
.43	.341735		.489261		.650181		.889049	2	.717467		.384354		.43
.44	.530284		.282279		.721305		.178618		.613394		.5961		.44
.45	.658233		.381212	2	.653776		.424824		.42093		.721426		.45
.46	.709687		.213522		.587103		.615932		.166339		.740361		.46
.47	.67773		.441564		.336623		.725241		.115098		.64843		.47
.48	.564406		.617324		.152009		.73642		.383399		.457684		.48
.49	.382587		.718122		.11559		.646437		.599731		.195289		.49
.5	.1527		.730202		.371435		.46603		.751951		.100182		.5
.51	.985552	2	.650633		.581657		.218611		.759404		.384281		.51
.52	.341448		.487897		.717707		.626524	2	.6762		.613282		.52
.53	.54669		.261091		.75023		.39165		.492468		.750873		.53
.54	.688932		2.22311	3	.701852		.572117		.233319		.771943		.54
.55	.749881		.26928		.548455		.727885		.643359	2	.67655		.55
.56	.720653		.506111		.318758		.782884		.357172		.471582		.56
.57	.603067		.681814		.421713	2	.727178		.601084		.188389		.57
.58	.40969		.772557		.544831		.566308		.758314		.129333		.58
.59	.1676		.764818		.503406		.320995		.803028		.431878		.59
.6	.109031		.657415		.697513		.246931	2	.725691		.670222		.6
.61	.372286		.462031		.798332		.280777		.535108		.804106		.61
.62	.52408		.202118		.791321		.55096		.257785		.80888		.62
.63	.746816		.897561	2	.672941		.745132		.654424	2	.680159		.63
.64	.808748		.375834		.457409		.8324		.385136		.435474		.64
.65	.77008		.617924		.172402		.796658		.6506		.112277		.65
.66	.632355		.782411		.143639		.637609		.817792		.237251		.66
.67	.411952		.844879		.446458		.581318		.856635		.554299		.67
.68	.137739		.793742		.692001		.580477	2	.756523		.783058		.68
.69	.170709		.632345		.842699		.282442		.529051		.880474		.69
.7	.460059		.37917	2	.731193		.587621		.207339		.824444		.7
.71	.597291		.661465		.774624		.808049		.158187		.61897		.71
.72	.843563		.265237		.956819		.905277		.507155		.295103		.72
.73	.893681		.568939		.247899		.858573		.778797		.28383	2	.73
.74	.819279		.800607		.108817		.669366		.922297		.475532		.74
.75	.632337		.923857		.460371		.362494		.906288		.780369		.75
.76	.352369		.915908		.751572		.161807	2	.725873		.94484		.76
.77	.136464		.771118		.933327		.405793		.403355		.929323		.77
.78	.333355		.505948		.37075		.739435		.319304	3	.726818		.78
.79	.639875		.152285		.849799		.355357		.427517		.367416		.79
.8	.899109		.240018		.581307		.100756		.786526		.840477	2	.8
.81	.101875		.612176		.201433		.676056		.100495		.537642		.81
.82	.994051		.903907		.231537		.572867		.102869		.894457		.82
.83	.819107		.106254		.645253		.145501		.612154		.106594		.83
.84	.509535		.105204		.963508		.337537		.456715		.999933		.84
.85	.102452		.860661		.111944		.772263		.482392	2	.688093		.85
.86	.346191		.505837		.106872		.107997		.573258		.18504		.86
.87	.76898		.353455	2	.801083		.116632		.996088		.399592		.87
.88	.102266		.476223		.347838		.992958		.120074		.917557		.88
.89	.12515		.935917		.16771		.56895		.110701		.12136		.89
.9	.119717		.12456		.791351		.549057	2	.699033		.116766		.9
.91	.91002		.151897		.121451		.687328		.461837	2	.740189		.91
.92	.40941		.110102		.13243		.121589		.691748		.949777		.92
.93	.241475		.527962		.11973		.143401		.128286		.813683	3	.93
.94	.931274		.154215		.617918		.117693		.147193		.141092		.94
.95	.15037		.397003		.2471		.454364		.106432		.143723		.95
.96	.1765		.16467		.12267		.62897		.462877	2	.673637		.96
.97	.137693		.183077		.18135		.146122		.126147		.738311		.97
.98	.17763		.112946		.1018		.137724		.202556		.19773		.98
.99	.15107		.913938		.11219		.214943		.710612		.114515		.99
1	.63232		.477709		.489541		.500193		.510184		.518895		1

NOTE: (The exponent p is 0 unless otherwise stated.)

Table 3-3. Eigenvalues for $c = 15\pi$ or aperture length = 15λ

n	λ_n	n	λ_n
0	.36515	18	-.36515
1	j.36515	19	-j.36515
2	-.36515	20	.36515
3	-j.36515	21	j.36514
4	.36515	22	-.36514
5	j.36515	23	-j.36513
6	-.36515	24	.36509
7	-j.36515	25	j.36493
8	.36515	26	-.36421
9	j.36515	27	-j.36054
10	-.36515	28	.34488
11	-j.36515	29	j.29765
12	.36515	30	-.21139
13	j.36515	31	-j.11750
14	-.36515	32	.04961
15	-j.36515	33	j.01863
16	.36515	34	-.00702
17	j.36515	35	

Table 3-4. Eigenfunctions for $c = 15\pi$ or aperture length = 15λ

$$\psi_n(x) = F_n(x) \times 10^{-p}$$

x	$F_0(x)$	p	$F_1(x)$	p	$F_2(x)$	p	$F_3(x)$	p	$F_4(x)$	p	$F_5(x)$	p	$F_6(x)$	p	$F_7(x)$	p	x
0	1.0000		0		1.3772		0		1.1821		0		1.0588		0		0
.01	1.0000		1.1888	84	1.3512		1.0214		1.1578		1.0444		1.0379		1.0444		.01
.02	1.0000		1.3748		1.3138		1.0472		1.0872		1.0465		1.0420		1.0103		.02
.03	1.0000		1.5578		1.2617		1.0763		1.0619		1.0427		1.0444		1.0348		.03
.04	1.0000		1.7325		1.1900		1.1100		1.0275		1.0478		1.0420		1.0485		.04
.05	1.0000		1.8984		1.0992		1.1500		0.9508		1.0418		1.0420		1.0485		.05
.06	1.0000		2.0666		1.0000		1.1900		0.8071		1.0418		1.0420		1.0485		.06
.07	1.0000		2.2379		0.8959		1.2300		0.6834		1.0418		1.0420		1.0485		.07
.08	1.0000		2.4127		0.7817		1.2700		0.5876		1.0418		1.0420		1.0485		.08
.09	1.0000		2.5912		0.6584		1.3100		0.5071		1.0418		1.0420		1.0485		.09
.10	1.0000		2.7735		0.5251		1.3500		0.4371		1.0418		1.0420		1.0485		.10
.11	1.0000		2.9596		0.3818		1.3900		0.3771		1.0418		1.0420		1.0485		.11
.12	1.0000		3.1496		0.2285		1.4300		0.3271		1.0418		1.0420		1.0485		.12
.13	1.0000		3.3435		0.0652		1.4700		0.2871		1.0418		1.0420		1.0485		.13
.14	1.0000		3.5412		-0.1985		1.5100		0.2571		1.0418		1.0420		1.0485		.14
.15	1.0000		3.7427		-0.4618		1.5500		0.2371		1.0418		1.0420		1.0485		.15
.16	1.0000		3.9480		-0.7251		1.5900		0.2271		1.0418		1.0420		1.0485		.16
.17	1.0000		4.1571		-1.0000		1.6300		0.2271		1.0418		1.0420		1.0485		.17
.18	1.0000		4.3700		-1.2871		1.6700		0.2371		1.0418		1.0420		1.0485		.18
.19	1.0000		4.5867		-1.5871		1.7100		0.2571		1.0418		1.0420		1.0485		.19
.20	1.0000		4.8072		-1.8984		1.7500		0.2871		1.0418		1.0420		1.0485		.20
.21	1.0000		5.0317		-2.2212		1.7900		0.3271		1.0418		1.0420		1.0485		.21
.22	1.0000		5.2602		-2.5565		1.8300		0.3771		1.0418		1.0420		1.0485		.22
.23	1.0000		5.4927		-2.9042		1.8700		0.4371		1.0418		1.0420		1.0485		.23
.24	1.0000		5.7292		-3.2643		1.9100		0.5071		1.0418		1.0420		1.0485		.24
.25	1.0000		5.9697		-3.6368		1.9500		0.5876		1.0418		1.0420		1.0485		.25
.26	1.0000		6.2142		-4.0217		1.9900		0.6834		1.0418		1.0420		1.0485		.26
.27	1.0000		6.4627		-4.4192		2.0300		0.7989		1.0418		1.0420		1.0485		.27
.28	1.0000		6.7152		-4.8293		2.0700		0.9300		1.0418		1.0420		1.0485		.28
.29	1.0000		6.9717		-5.2520		2.1100		1.0763		1.0418		1.0420		1.0485		.29
.30	1.0000		7.2322		-5.6873		2.1500		1.2371		1.0418		1.0420		1.0485		.30
.31	1.0000		7.4967		-6.1352		2.1900		1.4127		1.0418		1.0420		1.0485		.31
.32	1.0000		7.7652		-6.5957		2.2300		1.6000		1.0418		1.0420		1.0485		.32
.33	1.0000		8.0377		-7.0688		2.2700		1.7989		1.0418		1.0420		1.0485		.33
.34	1.0000		8.3142		-7.5545		2.3100		2.0071		1.0418		1.0420		1.0485		.34
.35	1.0000		8.5947		-8.0528		2.3500		2.2271		1.0418		1.0420		1.0485		.35
.36	1.0000		8.8792		-8.5637		2.3900		2.4571		1.0418		1.0420		1.0485		.36
.37	1.0000		9.1677		-9.0872		2.4300		2.6971		1.0418		1.0420		1.0485		.37
.38	1.0000		9.4602		-9.6233		2.4700		2.9471		1.0418		1.0420		1.0485		.38
.39	1.0000		9.7567		-10.1720		2.5100		3.2071		1.0418		1.0420		1.0485		.39
.40	1.0000		10.0572		-10.7323		2.5500		3.4771		1.0418		1.0420		1.0485		.40
.41	1.0000		10.3617		-11.3042		2.5900		3.7571		1.0418		1.0420		1.0485		.41
.42	1.0000		10.6692		-11.8877		2.6300		4.0471		1.0418		1.0420		1.0485		.42
.43	1.0000		10.9797		-12.4828		2.6700		4.3471		1.0418		1.0420		1.0485		.43
.44	1.0000		11.2932		-13.0895		2.7100		4.6571		1.0418		1.0420		1.0485		.44
.45	1.0000		11.6097		-13.7078		2.7500		4.9771		1.0418		1.0420		1.0485		.45
.46	1.0000		11.9292		-14.3378		2.7900		5.3071		1.0418		1.0420		1.0485		.46
.47	1.0000		12.2517		-14.9795		2.8300		5.6471		1.0418		1.0420		1.0485		.47
.48	1.0000		12.5772		-15.6330		2.8700		5.9971		1.0418		1.0420		1.0485		.48
.49	1.0000		12.9057		-16.2983		2.9100		6.3571		1.0418		1.0420		1.0485		.49
.50	1.0000		13.2372		-16.9755		2.9500		6.7271		1.0418		1.0420		1.0485		.50
.51	1.0000		13.5717		-17.6648		2.9900		7.1071		1.0418		1.0420		1.0485		.51
.52	1.0000		13.9092		-18.3663		3.0300		7.4971		1.0418		1.0420		1.0485		.52
.53	1.0000		14.2497		-19.0798		3.0700		7.8971		1.0418		1.0420		1.0485		.53
.54	1.0000		14.5932		-19.8053		3.1100		8.3071		1.0418		1.0420		1.0485		.54
.55	1.0000		14.9397		-20.5428		3.1500		8.7271		1.0418		1.0420		1.0485		.55
.56	1.0000		15.2892		-21.2923		3.1900		9.1571		1.0418		1.0420		1.0485		.56
.57	1.0000		15.6417		-22.0538		3.2300		9.5971		1.0418		1.0420		1.0485		.57
.58	1.0000		15.9972		-22.8273		3.2700		10.0471		1.0418		1.0420		1.0485		.58
.59	1.0000		16.3557		-23.6128		3.3100		10.5071		1.0418		1.0420		1.0485		.59
.60	1.0000		16.7172		-24.4103		3.3500		10.9771		1.0418		1.0420		1.0485		.60
.61	1.0000		17.0817		-25.2198		3.3900		11.4571		1.0418		1.0420		1.0485		.61
.62	1.0000		17.4492		-26.0423		3.4300		11.9471		1.0418		1.0420		1.0485		.62
.63	1.0000		17.8197		-26.8768		3.4700		12.4471		1.0418		1.0420		1.0485		.63
.64	1.0000		18.1932		-27.7233		3.5100		12.9571		1.0418		1.0420		1.0485		.64
.65	1.0000		18.5697		-28.5818		3.5500		13.4771		1.0418		1.0420		1.0485		.65
.66	1.0000		18.9492		-29.4523		3.5900		13.9971		1.0418		1.0420		1.0485		.66
.67	1.0000		19.3317		-30.3348		3.6300		14.5271		1.0418		1.0420		1.0485		.67
.68	1.0000		19.7172		-31.2293		3.6700		15.0671		1.0418		1.0420		1.0485		.68
.69	1.0000		20.1057		-32.1358		3.7100		15.6171		1.0418		1.0420		1.0485		.69
.70	1.0000		20.4972		-33.0543		3.7500		16.1771		1.0418		1.0420		1.0485		.70
.71	1.0000		20.8917		-33.9848		3.7900		16.7471		1.0418		1.0420		1.0485		.71
.72	1.0000		21.2892		-34.9273		3.8300		17.3271		1.0418		1.0420		1.0485		.72
.73	1.0000		21.6897		-35.8818		3.8700		17.9171		1.0418		1.0420		1.0485		.73
.74	1.0000		22.0932		-36.8483		3.9100		18.5171		1.0418		1.0420		1.0485		.74
.75	1.0000		22.4997		-37.8268		3.9500		19.1271		1.0418		1.0420		1.0485		.75
.76	1.0000		22.9092		-38.8173		3.9900		19.7471		1.0418		1.0420		1.0485		.76
.77	1.0000		23.3217		-39.8198		4.0300		20.3771		1.0418		1.0420		1.0485		.77
.78	1.0000		23.7372		-40.8343		4.0700		21.0171		1.0418		1.0420		1.0485		.78
.79	1.0000		24.1557		-41.8608		4.1100		21.6671		1.0418		1.0420		1.0485		.79
.80	1.0000		24.5772		-42.8993		4.1500		22.3271		1.0418		1.0420		1.0485		.80
.81	1.0000		24.9997		-43.9498		4.1900		22.9971		1.0418		1.0420		1.0485		.81
.82	1.0000		25.4242		-45.0123		4.2300		23.6771		1.0418		1.0420		1.0485		.82
.83	1.0000		25.8517		-46.0868		4.2700		24.3671		1.0418		1.0420		1.0485		.83
.84	1.0000		26.2812		-47.1733		4.3100		25.0671		1.0418		1.0420		1.0485		.84
.85	1.0000		26.7127		-48.2718		4.3500		25.7771		1.0418		1.0420		1.0485		

TABLE 3-4. (continued)

x	F ₈ (x)	p	F ₉ (x)	p	F ₁₀ (x)	p	F ₁₁ (x)	p	F ₁₂ (x)	p	F ₁₃ (x)	p	F ₁₄ (x)	p	F ₁₅ (x)	p	x
0	.72556		0		.922278		0		.871901		0		.3345		0		0
.01	.725076		.749574		.925459		.28639		.850363		.287415		.702855		.204029		.01
.02	.724582		.752843		.928639		.56497		.802886		.54152		.63434		.54157		.02
.03	.724093		.756104		.931819		.84355		.725287		.802886		.407264		.72375		.03
.04	.723604		.759365		.935000		.118927		.603475		.839296		.12967		.410302		.04
.05	.723115		.762626		.938181		.402027	2	.903645	2	.848469		.164153		.790417		.05
.06	.722626		.765887		.941362		.680689		.823383		.759317		.438047		.666336		.06
.07	.722137		.769148		.944543		.957393		.717123		.58215		.658156		.459623		.07
.08	.721648		.772409		.947724		.121986		.510201		.337353		.797353		.134737		.08
.09	.721159		.775670		.950905		.401147		.25148		.870841		.532009	2	.838573		.09
.1	.720670		.778931		.954086		.680689		.510201		.870841		.532009	2	.838573		.1
.11	.720181		.782192		.957267		.957393	2	.871901		.287415		.702855		.204029		.11
.12	.719692		.785453		.960448		.121986		.510201		.337353		.797353		.134737		.12
.13	.719203		.788714		.963629		.401147		.25148		.870841		.532009	2	.838573		.13
.14	.718714		.791975		.966810		.680689		.510201		.870841		.532009	2	.838573		.14
.15	.718225		.795236		.970000		.957393	2	.871901		.287415		.702855		.204029		.15
.16	.717736		.798497		.973181		.121986		.510201		.337353		.797353		.134737		.16
.17	.717247		.801758		.976362		.401147		.25148		.870841		.532009	2	.838573		.17
.18	.716758		.805019		.979543		.680689		.510201		.870841		.532009	2	.838573		.18
.19	.716269		.808280		.982724		.957393	2	.871901		.287415		.702855		.204029		.19
.2	.715780		.811541		.985905		.121986		.510201		.337353		.797353		.134737		.2
.21	.715291		.814802		.989086		.401147		.25148		.870841		.532009	2	.838573		.21
.22	.714802		.818063		.992267		.680689		.510201		.870841		.532009	2	.838573		.22
.23	.714313		.821324		.995448		.957393	2	.871901		.287415		.702855		.204029		.23
.24	.713824		.824585		.998629		.121986		.510201		.337353		.797353		.134737		.24
.25	.713335		.827846		.100000		.401147		.25148		.870841		.532009	2	.838573		.25
.26	.712846		.831107		.100000		.680689		.510201		.870841		.532009	2	.838573		.26
.27	.712357		.834368		.100000		.957393	2	.871901		.287415		.702855		.204029		.27
.28	.711868		.837629		.100000		.121986		.510201		.337353		.797353		.134737		.28
.29	.711379		.840890		.100000		.401147		.25148		.870841		.532009	2	.838573		.29
.3	.710890		.844151		.100000		.680689		.510201		.870841		.532009	2	.838573		.3
.31	.710401		.847412		.100000		.957393	2	.871901		.287415		.702855		.204029		.31
.32	.709912		.850673		.100000		.121986		.510201		.337353		.797353		.134737		.32
.33	.709423		.853934		.100000		.401147		.25148		.870841		.532009	2	.838573		.33
.34	.708934		.857195		.100000		.680689		.510201		.870841		.532009	2	.838573		.34
.35	.708445		.860456		.100000		.957393	2	.871901		.287415		.702855		.204029		.35
.36	.707956		.863717		.100000		.121986		.510201		.337353		.797353		.134737		.36
.37	.707467		.866978		.100000		.401147		.25148		.870841		.532009	2	.838573		.37
.38	.706978		.870239		.100000		.680689		.510201		.870841		.532009	2	.838573		.38
.39	.706489		.873500		.100000		.957393	2	.871901		.287415		.702855		.204029		.39
.4	.706000		.876761		.100000		.121986		.510201		.337353		.797353		.134737		.4
.41	.705511		.880022		.100000		.401147		.25148		.870841		.532009	2	.838573		.41
.42	.705022		.883283		.100000		.680689		.510201		.870841		.532009	2	.838573		.42
.43	.704533		.886544		.100000		.957393	2	.871901		.287415		.702855		.204029		.43
.44	.704044		.889805		.100000		.121986		.510201		.337353		.797353		.134737		.44
.45	.703555		.893066		.100000		.401147		.25148		.870841		.532009	2	.838573		.45
.46	.703066		.896327		.100000		.680689		.510201		.870841		.532009	2	.838573		.46
.47	.702577		.899588		.100000		.957393	2	.871901		.287415		.702855		.204029		.47
.48	.702088		.902849		.100000		.121986		.510201		.337353		.797353		.134737		.48
.49	.701599		.906110		.100000		.401147		.25148		.870841		.532009	2	.838573		.49
.5	.701110		.909371		.100000		.680689		.510201		.870841		.532009	2	.838573		.5
.51	.700621		.912632		.100000		.957393	2	.871901		.287415		.702855		.204029		.51
.52	.700132		.915893		.100000		.121986		.510201		.337353		.797353		.134737		.52
.53	.699643		.919154		.100000		.401147		.25148		.870841		.532009	2	.838573		.53
.54	.699154		.922415		.100000		.680689		.510201		.870841		.532009	2	.838573		.54
.55	.698665		.925676		.100000		.957393	2	.871901		.287415		.702855		.204029		.55
.56	.698176		.928937		.100000		.121986		.510201		.337353		.797353		.134737		.56
.57	.697687		.932198		.100000		.401147		.25148		.870841		.532009	2	.838573		.57
.58	.697198		.935459		.100000		.680689		.510201		.870841		.532009	2	.838573		.58
.59	.696709		.938720		.100000		.957393	2	.871901		.287415		.702855		.204029		.59
.6	.696220		.941981		.100000		.121986		.510201		.337353		.797353		.134737		.6
.61	.695731		.945242		.100000		.401147		.25148		.870841		.532009	2	.838573		.61
.62	.695242		.948503		.100000		.680689		.510201		.870841		.532009	2	.838573		.62
.63	.694753		.951764		.100000		.957393	2	.871901		.287415		.702855		.204029		.63
.64	.694264		.955025		.100000		.121986		.510201		.337353		.797353		.134737		.64
.65	.693775		.958286		.100000		.401147		.25148		.870841		.532009	2	.838573		.65
.66	.693286		.961547		.100000		.680689		.510201		.870841		.532009	2	.838573		.66
.67	.692797		.964808		.100000		.957393	2	.871901		.287415		.702855		.204029		.67
.68	.692308		.968069		.100000		.121986		.510201		.337353		.797353		.134737		.68
.69	.691819		.971330		.100000		.401147		.25148		.870841		.532009	2	.838573		.69
.7	.691330		.974591		.100000		.680689		.510201		.870841		.532009	2	.838573		.7
.71	.690841		.977852		.100000		.957393	2	.871901		.287415		.702855		.204029		.71
.72	.690352		.981113		.100000		.121986		.510201		.337353		.797353		.134737		.72
.73	.689863		.984374		.100000		.401147		.25148		.870841		.532009	2	.838573		.73
.74	.689374		.987635		.100000		.680689		.510201		.870841		.532009	2	.838573		.74
.75	.688885		.990896		.100000		.957393	2	.871901		.287415		.702855		.204029		.75
.76	.688396		.994157		.100000		.121986		.510201		.337353		.797353		.134737		.76
.77	.687907		.997418		.100000		.401147		.25148		.870841		.532009	2	.838573		.77
.78	.687418		.100679		.100000		.680689		.510201		.870841		.532009	2	.838573		.78
.79	.686929		.103940		.100000		.957393	2	.871901		.287415		.702855		.204029		.79
.8	.686440		.107201		.100000		.121986		.510201		.337353		.797353		.134737		.8
.81	.685951		.110462		.100000		.401147		.25148		.870841		.532009	2	.838573		.81
.82	.685462		.113723		.100000		.680689		.510201		.870841		.532009	2	.838573		.82
.83	.684973		.116984		.100000		.957393	2	.871901		.287415		.702855		.204029		.83
.84	.684484		.120245		.100000		.121986		.510201		.337353		.797353		.134737		.84
.85	.683995		.123506		.100000		.401147		.25148		.870841		.532009	2	.838573		.85
.86	.683506		.126767		.100000		.680689		.510201		.870841		.532009	2	.838573		.86
.87</																	

TABLE 3-4. (continued)

x	F ₁₀ (x)	p	F ₇ (x)	p	F ₁₀ (x)	p	F ₁₀ (x)	p	F ₂₀ (x)	p	F ₂₁ (x)	p	F ₂₂ (x)	p	F ₂₃ (x)	p	x
0	.79771		0		.75936		0		.725583		0		.69148		0		0
.01	.74766		.251563		.701786		.290915		.665205		.28834		.629203		.283442		.01
.02	.58748		.541068		.536424		.55346		.494101		.326869		.45466		.314395		.02
.03	.34477		.711038		.288398		.694497		.24469		.674362		.198236		.650075		.03
.04	.14477	2	.77788		.274912	3	.694497		.24469		.674362		.198236	2	.650075		.04
.05	.237083		.151336		.204313		.672416		.337963	2	.61433		.369401		.557224		.05
.06	.456248		.578218		.41211		.494628		.367089		.417017		.578962		.345724		.06
.07	.696235		.340701		.705762		.37751		.702145		.14737		.685061		.698832	2	.07
.08	.797439		.532532		.6282		.74952	2	.72634		.148131		.668521		.2193		.08
.09	.771755		.224241		.703598		.344029		.619365		.418517		.532185		.468314		.09
.1	.55597		.503334		.536998		.576224		.412001		.617109		.360284		.631017		.1
.11	.459134		.691783		.288247		.717157		.141299		.709556		.141441	2	.67717		.11
.12	.177483		.780241		.494317	3	.744215		.15647		.679756		.275167		.598025		.12
.13	.120527		.755836		.278104		.69259		.429019		.532632		.515962		.407988		.13
.14	.432545		.621952		.74676		.497663		.631183		.293293		.665128		.141941		.14
.15	.330737		.397765		.718156		.189139		.729387		.273519		.695839		.151128		.15
.16	.772151		.711546		.711546		.110185		.77191		.60254		.409994		.471759		.16
.17	.961117		.144436		.714117		.56959		.56959		.23234		.684837		.606973		.17
.18	.735741		.498891		.581141		.614759		.334492		.72029		.127592		.685335		.18
.19	.56177		.668389		.298728		.740107		.449151	2			.16935		.637465		.19
.2	.311163	2	.782932		.354064	3	.749132		.253215		.632291		.438197		.471785		.2
.21	.166891		.782932		.354064		.640117		.253215		.43538		.629537		.218415		.21
.22	.280731		.67673		.429879		.63784		.163333		.709811		.709811		.076324		.22
.23	.542		.470614		.721528		.151117		.78872		.13808		.664259		.358384		.23
.24	.731667		.196604		.787856		.152719		.685504		.417384		.500636		.575889		.24
.25	.824224		.166889		.736953		.454215		.56974		.626916		.247349		.688632		.25
.26	.40713		.396546		.570051		.64939		.259032		.730762		.511189	2	.675497		.26
.27	.582437		.632155		.328682		.764529		.426218	2	.710861		.342223		.538325		.27
.28	.466434		.780333		.511507	2	.761404		.359115		.570085		.574562		.301653		.28
.29	.187424		.82013		.2728		.640074		.582521		.331749		.706922		.828252	3	.29
.3	.110567		.745704		.538531		.418854		.732643		.357115	2	.715532		.288479		.3
.31	.410126		.567057		.726951		.131463		.7654		.268337		.598267		.534498		.31
.32	.650673		.308588		.810264		.178178		.674791		.329226		.375053		.684665		.32
.33	.80842		.563222	3	.775961		.462693		.474822		.702884		.844241	2	.711104		.33
.34	.862805		.500345		.628885		.67849		.197052		.797934		.22317		.608285		.34
.35	.80661		.567637		.389006		.792393		.114467		.689638		.494133		.394082		.35
.36	.646743		.799773		.390946		.786648		.410211		.503601		.681042		.106817		.36
.37	.403257		.390946		.221928		.66157		.645061		.231955		.750679		.202281		.37
.38	.106606		.673392		.506191		.435449		.775758		.806683	2	.690788		.477798		.38
.39	.206114		.653042		.720929		.141746		.786782		.382712		.510322		.66917		.39
.4	.49604		.464982		.835484		.177994		.673737		.6242		.239244		.741648		.4
.41	.727293		.173049		.833311		.470462		.453558		.734958		.453558	2	.681328		.41
.42	.871378		.144624		.714201		.699006		.160118		.781201		.379443		.498006		.42
.43	.91057		.446671		.494201		.82467		.161361		.669425		.622184		.223509		.43
.44	.839212		.693899		.203281		.831235		.461129		.446972		.760911		.939054	2	.44
.45	.667620		.674325		.118903		.716134		.692955		.149211		.771161		.398066		.45
.46	.41335		.307132		.428509		.495465		.820561		.176191		.650158		.63482		.46
.47	.108433		.845421		.683502		.200352		.823797		.476953		.417513		.761611		.47
.48	.213484		.676413		.84931		.127231		.701318		.704574		.110981		.755021		.48
.49	.514886		.420819		.903374		.44073		.470784		.822071		.21755		.61495		.49
.5	.761548		.109826		.838801		.69581		.166123		.809846		.51456		.364711	2	.5
.51	.925778		.218852		.661248		.366111		.16762		.668759		.729041		.470293		.51
.52	.789598		.525674		.395573		.39874		.48104		.419947		.824965		.28333		.52
.53	.745526		.774054		.748109		.817077		.727692		.101471		.785506		.569033		.53
.54	.798421		.93465		.567783		.621599		.870897		.237659		.615249		.760074		.54
.55	.964228		.398642		.358072		.538072		.889011		.345134		.345134		.822412		.55
.56	.267171		.929662		.425864		.425864	3	.776441		.773258		.716217	3	.74382		.56
.57	.263464		.764203		.808966		.335714		.254038		.886509		.333755		.256176		.57
.58	.391302		.910499		.984828		.637037		.246664		.866577		.626463		.253214		.58
.59	.687506		.196156		.897444		.860222		.101043		.71517		.823578		.114847		.59
.6	.922775		.145206		.701763		.976095		.441047		.453605		.792312		.449864		.6
.61	.107551		.477791		.420203		.999343		.726478		.119506		.821162		.715462		.61
.62	.11324		.767517		.846764	2	.840159		.918064		.258863		.618769		.866409		.62
.63	.108223		.989494		.26737		.603436		.989324		.569717		.31555		.876109		.63
.64	.35069		.111074		.597378		.288591		.329883		.82325		.435125	2	.740976		.64
.65	.729358		.113182		.869989		.684176	2	.746534		.968408		.403636		.481009		.65
.66	.443557		.104763		.105676		.42561		.42561		.977967		.710503		.13654		.66
.67	.117219		.866493		.113887		.742175		.111521		.851246		.917452		.238283		.67
.68	.226447		.605034		.11086		.982631		.261715		.604161		.992529		.584153		.68
.69	.563214		.289841		.969451		.112073		.612412		.268696		.623029		.845982		.69
.7	.871807		.650931	2	.735122		.141487		.898667		.111726		.717086		.981493		.7
.71	.113502		.421043		.427332		.104411		.108691		.488251		.402214		.967682		.71
.72	.134059		.757057		.731502	2	.637617		.11555		.813079		.210816	2	.80419		.72
.73	.148207		.105195		.298125		.54277		.109667		.104551		.374727		.513149		.73
.74	.155776		.128322		.657364		.187317		.516671		.115679		.131382		.152617		.74
.75	.157101		.146076		.981223		.124314		.634185		.11331		.100303		.27472		.75
.76	.15289		.158102		.124314		.578349		.277451		.976534		.115192		.699441		.76
.77	.144118		.159291		.144554		.927349		.119536		.703941		.115866		.964296		.77
.78	.131508		.156184		.156727		.122036		.52115		.344048		.102108		.114576		.78
.79	.117424		.147912		.161384		.144083		.894056		.666819	2	.742427		.117762		.79
.8	.101773		.155654		.159147		.158032		.121048		.466756		.367589		.105342		.8
.81	.859529		.120708		.151074		.16379		.145051		.884239		.557869	2	.786812		.81
.82	.70732		.104329		.158498		.161963		.160325		.122189		.483552		.4082		.82
.83	.56756		.876389		.122857		.153683		.16668		.14775		.901716		.359573	2	.83
.84	.444481		.715588		.105601		.140423		.164732		.16383		.125754		.507735		.84
.85	.336127		.567781		.87574		.12381		.155726		.170167		.152424		.948415		.85
.86	.248341		.437478		.710326		.105448		.141321		.167442		.168681		.132004		.86
.87	.178594		.326979		.555548		.867748		.123359		.157074		.174246		.159239		.87

TABLE 3-4. (continued)

x	F ₂₄ (x)	p	F ₂₅ (x)	p	F ₂₆ (x)	p	F ₂₇ (x)	p	F ₂₈ (x)	p	F ₂₉ (x)	p	F ₃₀ (x)	p	F ₃₁ (x)	p	x
0	.655575		0		.613438		0		.556237		0		.525451		0		0
.01	.592884		.579198		.55141		.560006		.497351		.40249		.471221		.262992		.01
.02	.416779		.496191		.377847		.465332		.333667		.245947		.30033		.467147		.02
.03	.160881		.619604		.12778		.576256		.918608		.245947		.7.33368		.566946		.03
.04	-.125946		.621074		-.148268		.566525		-.157648		.507181		-.168258		.540019		.04
.05	-.388912		.500235		.354555		.439475		-.350274		.380627		-.37503		.392347		.05
.06	-.577751		.28076		.561274		.221123		-.52557		.170992		-.506605		.156906		.06
.07	-.65631		.5.73334		-.614551		.4.33346		-.554272		-.7.5443		-.52284		-.113671		.07
.08	-.669455		-.270813		-.543762		-.299181		-.468571		-.302923		-.42401		-.35883		.08
.09	-.445975		-.494482		-.562753		-.493413		-.283426		-.470657		-.231699		-.52361		.09
.1	-.146922		-.621137		-.108046		-.585701		-.3.78738		-.533562		1.19802		-.570934		.1
.11	9.02851	2	-.620873		.165012		-.556744		.216203		-.481.74		.253514		-.489926		.11
.12	.360878		-.507357		.412553		-.412295		.2502		-.325678		.43976		-.298397		.12
.13	.563171		-.288693		.573225		-.182015		.544255		-.098591		.529605		-.3.89265		.13
.14	.62841		-.126378	2	.618401		8.66097	2	.548438		.150415		.503041		.230992		.14
.15	.628171		.266697		.338663		.33809		.435374		.367608		.36528		.449825		.15
.16	.477901		.694432		.340773		.520188		.231461		.505376		.147151		.569631		.16
.17	.235919		.626687		8.35537		.595099		-.2.31945		.535875		-.104421		.562801		.17
.18	-.5.19776	2	.634352		-.185682		.546928		-.273826		.449085		-.333853		.430908		.18
.19	-.330949		-.518441		-.43163		.389225		-.467346		.265335		-.490571		.201999		.19
.2	-.547989		.309165		-.58733		.143051		-.582558		2.224436	2	-.539753		-.7.18095	2	.2
.21	-.661528		-.2.20165	2	-.625045		-.129778		.538915		-.470134		-.529388		-.529388		.21
.22	-.645959		-.261682		-.536699		-.376923		-.409926		-.424969		-.296537		-.513519		.22
.23	-.513525		-.495346		-.335925		-.547132		-.17751		-.534435		-.056735		-.581793		.23
.24	-.279833		-.632969		-.7.29662	2	-.604845		.084981		-.528796		.196749		-.518517		.24
.25	8.58013	3	-.647175		.267742		-.537592		.3306		-.408703		.408121		-.337165		.25
.26	.236973		-.534609		.451657		-.358664		.507462		-.199522		.530566		-.7.77703	2	.26
.27	.530609		-.31668		.603852		-.104481		.57776		.5.40096	2	.536602		.201899		.27
.28	.664259		-.3.54664	2	.635215		.172816		.526022		.297333		.424225		.439187		.28
.29	.674088		.294408		.538772		.416044		.362455		.477784		.217503		.580566		.29
.3	.555602		.496377		.333194		.574744		.121112		.555959		-.3.85557	2	.593702		.3
.31	.331153		.64294		.5.91673	2	.615612		-.147532		.5143		-.2878		.47485		.31
.32	.4.26163	2	.664942		-.228658		.529459		-.386769		.361014		-.475178		.24989		.32
.33	-.256085		.557322		.333229		.545783		.123469		-.558905		-.031259		-.031259		.33
.34	-.5087		.340204		-.623264		6.66665	2	-.550366		-.133621		-.519738		-.305673		.34
.35	-.657396		.5.51056	2	-.649899		-.215733		.510249		-.363802		-.365382		-.511445		.35
.36	.70172		-.243016		-.546227		-.455839		.321427	2	-.326006		-.129006		-.601604		.36
.37	-.604375		-.496347		-.332055		-.603824		-.6.25693		-.570609		.137862		-.554796		.37
.38	-.332618		-.653393		-.4.9424	2	-.628453		.211049		-.492013		.376532		-.389306		.38
.39	-.105427		-.688596		.245642		-.523635		.442637		-.306158		.554023		-.116255		.39
.4	.204241		-.584526		.494237		-.309848		.583035		-.5.22708	2	.574783		.178707		.4
.41	-.478587		-.373445		.546282	2	-.3.01332		.601188		.215369		.486695		.438392		.41
.42	-.660466		8.33351	2	.670561		.228569		.492311		.439908		.293544		.603896		.42
.43	.731905		.547825		.551061		.496979		.278164		.569836		.513783	2	.63694		.43
.44	-.66175		.493386		.338347		.63561		2.30634	3	.578599		-.240523		.528717		.44
.45	-.467914		.66919		.4.56687	2	-.644973		-.75531		.462025		-.4621		.30421		.45
.46	.185027		.719196		-.259448		.52181		-.4.97706		.244152		-.583749		.7.37994	3	.46
.47	-.135459		.630855		-.516487		.28994		-.620424		-.2.92586	2	-.577372		-.28977		.47
.48	-.434822		.420276		-.673896		-.4.31644	3	-.611499		-.22965		-.442837		-.52189		.48
.49	-.657804		.12645		-.69935		-.301379		-.473269		-.50825		-.208343		-.635463		.49
.5	-.76267		-.192418		-.586294		-.540485		-.233283	2	-.609859		.7.54435	2	-.603079		.5
.51	-.728966		-.484828		-.355453		-.6.72006		.5.91885		-.530402		.346284		-.429989		.51
.52	-.561319		-.684784		-.5.09968	2	-.667692		.343232		-.424733		.543952		-.153387		.52
.53	-.288736		-.757789		.267935		-.52664		.558932		-.174766		.623563		.165526		.53
.54	.4.05092	2	-.687878		.538697		-.275774		.659958		.116302		.56571		.455021		.54
.55	.36749		-.486442		.70738		.3.55141	2	.623336		.387531		.381129		.648914		.55
.56	.653166		-.189743		.73944		.344151		.454765		.57855		.108797		.701649		.56
.57	.788967		.147681		.626632		.587885		.187595		.547279		-.192238		.598977		.57
.58	.209524		.463076		.368887		.715989		-.123642		.576833		-.435574		.361742		.58
.59	.677956		.697091		.7.07279	2	.700617		-.414349		.380066		-.622012		.041796		.59
.6	.42706	2	.804843		-.267101		.542514		-.623194		9.72853	2	-.652776		-.289186		.6
.61	9.53875		.764345		-.559218		.271235		-.704941		-.211949		-.538464		-.555603		.61
.62	-.225734		.980747		.748169		-.6.03962	2	-.640128		-.481289		-.301686		-.695037		.62
.63	-.57635		.285628		.795282		-.386564		-.439526		-.631727		.7.31098	3	-.672743		.63
.64	.798587		-.6.45735	2	.689756		-.641455		.142491		-.684404		.421193		-.490164		.64
.65	-.886345		-.417958		-.448209		.772409		.191106		-.569339		.570093		-.185437		.65
.66	-.822236		-.69886		-.114693		-.750617		.452162		-.328145		.697015		-.17444		.66
.67	-.614736		-.858472		.248813		-.577227		.697268		-.1.00483	2	.670754		.508184		.67
.68	-.29664		-.865982		.573613		-.283585		.761477		.318065		.493419		.738441		.68
.69	8.02094	2	-.717025		.797173		.7.44703	2	.668081		.58558		.260784		.809581		.69
.7	.453647		-.43497		.87495		.42722		.432596		.733176		-.144772		.701049		.7
.71	.761465		-.6.71354	2	.788945		.704541		.100242		.725778		-.46738		.433095		.71
.72	.951634		.323415		.551396		.84965		-.226692		.560652		-.69412		.6.33799	2	.72
.73	.390774		.668749		.202897		.83043		-.581561		.268822		-.771265		-.325978		.73
.74	.869491		.907779		-.124711		.646134		-.79034		-.090635		-.676774		-.642144		.74
.75	.603752		.996762		.569378		.323113		-.841492		-.442386		-.426178		-.811063		.75
.76	.232435		.916797		-.851803		-.6.56152	2	.71989		-.710378		-.7.03774	2	-.786642		.76
.77	-.189266		.677081		-.987952		-.460979		-.444737		-.833963		.31441		-.566891		.77
.78	-.598074		.313432		-.548891		-.781661		-.6.70041	2	-.781227		.642624		-.195234		.78
.79	-.932468		-.117593		-.736178		-.963509		.339957		-.596684		.837953		.247608		.79
.8	-.1.14174		-.547892		-.381895		-.966842		.694445		-.201575		.850607		.660635		.8
.81	-.1.18312		-.908739		.5.66493	2	-.784338		.922493		.213487		.659945		.945043		.81
.82	-.1.37601		-.1.10158		.703521		-.443613		.572551		.692224		.326183		.1.0216		.82
.83	-.802644		-.1.20691		.89154		.7.53682		.826517		.830097		-.109635		.877607		.83
.84	-.405471		-.1.08303		.1.14464		.467609		.504328		.981961		-.54				

TABLE 3-4. (continued)

x	F ₃₂ (x)	p	F ₃₃ (x)	p	F ₃₄ (x)	p	F ₃₅ (x)	p	x
0	.571563		0		.68781		0		0
.01	.505392		.279291		.536112		.296253		.01
.02	.322012		.453314		.355466		.519946		.02
.03	.632796	2	.592022		.054789		.616261		.03
.04	.211697		.552272		.238826		.561333		.04
.05	.440075		.383237		.437146		.369038		.05
.06	.260778		.124293		.557967		.858766	2	.06
.07	.440456		.164135		.576656		.218697		.07
.08	.442432		.414691		.417213		.470006		.08
.09	.214445		.268612		.177187		.606319		.09
.1	.613524	2	.589837		.141336		.593972		.1
.11	.322077		.473103		.406923		.435675		.11
.12	.507965		.245434		.570858		.169945		.12
.13	.576275		.401264	2	.677402		.138237		.13
.14	.51105		.316846		.443613		.413278		.14
.15	.326838		.519861		.261351		.587477		.15
.16	.654569	2	.60135		.340349	2	.617673		.16
.17	.213433		.541784		.321836		.499932		.17
.18	.446005		.354603		.532964		.251648		.18
.19	.578756		.832059	2	.51634		.553119	2	.19
.2	.580848		.209086		.551409		.350133		.2
.21	.451251		.453786		.353127	2	.559524		.21
.22	.21912		.593172		.555734		.631625		.22
.23	.627379	2	.594064		.234173		.547949		.23
.24	.329884		.45254		.440312		.528336		.24
.25	.520746		.209452		.615732		.263077	2	.25
.26	.501161		.868903	2	.601617		.283918		.26
.27	.5242		.364012		.444433		.525608		.27
.28	.334427		.556916		.175876		.638461		.28
.29	.647251	2	.618484		.134510		.593646		.29
.3	.223363		.53442		.413541		.401194		.3
.31	.463641		.32312		.594564		.107835		.31
.32	.600426		.355224	2	.631921		.214266		.32
.33	.601441		.26658		.515858		.485189		.33
.34	.465426		.506339		.273294		.637035		.34
.35	.222682		.628634		.579424	2	.630933		.35
.36	.071626		.603493		.342827		.467013		.36
.37	.349932		.43562		.567163		.184789		.37
.38	.547819		.163532		.655594		.146259		.38
.39	.618710		.149182		.565268		.443694		.39
.4	.545012		.428649		.371759	2	.63258		.4
.41	.342318		.608096		.657266		.664339		.41
.42	.562547	2	.643768		.258996		.522668		.42
.43	.247751		.52587		.525128		.259498		.43
.44	.499411		.279822		.661236		.785094	2	.44
.45	.35965		.360214	2	.637923		.400178		.45
.46	.63471		.347775		.456923		.623913		.46
.47	.48388		.580739		.16076		.691716		.47
.48	.250436		.678161		.179107		.584258		.48
.49	.555501	2	.614913		.479394		.52642		.49
.5	.591229		.403976		.665312		.182625	2	.5
.51	.597225		.593466	2	.689151		.363063		.51
.52	.664085		.242022		.542544		.619741		.52
.53	.574058		.523347		.259249		.721045		.53
.54	.349761		.681242		.924142	2	.638347		.54
.55	.030456		.675493		.42586		.389791		.55
.56	.299337		.504636		.657403		.366372	2	.56
.57	.566408		.207179		.727177		.331559		.57
.58	.706835		.146984		.614617		.619431		.58
.59	.689229		.472771		.3445		.75037		.59
.6	.503624		.690215		.181955	2	.6869		.6
.61	.201798		.744011		.383647		.441476		.61
.62	.191241		.617559		.659195		.744874	2	.62
.63	.472839		.337985		.772696		.31974		.63
.64	.683863		.293666	2	.691378		.637393		.64
.65	.737059		.395899		.431316		.792784		.65
.66	.610275		.670861		.548055	2	.739341		.66
.67	.33129		.78362		.344116		.486039		.67
.68	.037118		.7017		.662768		.954949	2	.68
.69	.409022		.440037		.815979		.33055		.69
.7	.695103		.588584	2	.758751		.4677167		.7
.71	.823895		.349722		.40946		.846855		.71
.72	.759444		.683531		.997882	2	.786863		.72
.73	.51125		.855548		.339207		.505647		.73
.74	.133547		.816395		.701975		.734256	2	.74
.75	.295842		.568407		.038371		.394231		.75
.76	.645545		.167356		.84103		.766309		.76
.77	.894534		.289204		.56304		.932363		.77
.78	.894741		.684799		.170199		.635064		.78
.79	.636812		.91296		.372966		.489937		.79
.8	.244879		.905306		.781575		.142336	2	.8
.81	.231401		.691608		.986106		.556138		.81
.82	.67629		.206207		.916955		.91938		.82
.83	.075022		.321842		.577173		.103806		.83
.84	.104256		.793645		.481068	2	.837418		.84
.85	.010701		.137322		.527223		.557424		.85
.86	.420403		.107405		.97057		.269351		.86
.87	.14180		.768259		.116042		.849689		.87
.88	.678775		.219916		.74674		.118368		.88
.89	.110799		.432521		.483706		.113904		.89
.9	.122789		.100345		.225133		.663061		.9
.91	.5216		.130633		.93716		.978529	2	.91
.92	.344654		.10795		.136401		.901278		.92
.93	.237284		.67637		.134055		.142608		.93
.94	.112719		.186641		.77306		.138697		.94
.95	.16802		.115803		.23007		.664431		.95
.96	.177497		.135703		.133575		.570734		.96
.97	.118896		.185126		.230225		.177303		.97
.98	.160403		.839196		.152223		.201091		.98
.99	.202333		.146368		.707185		.504017	2	.99
1	.47361		.510137		.55312		.546191		1

NOTE: (The exponent p is 0 unless otherwise stated.)

Table 3-5. Eigenvalues for $c = 20\pi$ or aperture length $\approx 20\lambda$

n	λ_n	n	λ_n
0	.31623	23	-j.31623
1	j.31623	24	.31623
2	-.31623	25	j.31623
3	-j.31623	26	-.31623
4	.31623	27	-j.31623
5	j.31623	28	.31623
6	-.31623	29	j.31623
7	-j.31623	30	-.31622
8	.31623	31	-j.31622
9	j.31623	32	.31622
10	-.31623	33	j.31620
11	-j.31623	34	-.31619
12	.31623	35	-j.31593
13	j.31623	36	.31530
14	-.31623	37	j.31142
15	-j.31623	38	-.29795
16	.31623	39	-j.25692
17	j.31623	40	.17559
18	-.31623	41	j.09214
19	-j.31623	42	-.03767
20	.31623	43	-j.01415
21	j.31623	44	.00532
22	-.31623	45	j.00208

Table 3-6. Eigenfunctions for $c = 20\pi$ or aperture length $= 20\lambda$

$$\psi_n(x) = F_n(x) \times 10^{-p}$$

x	$F_0(x)$	p	$F_1(x)$	p	$F_2(x)$	p	$F_3(x)$	p	$F_4(x)$	p	$F_5(x)$	p	$F_6(x)$	p	$F_7(x)$	p	x
0	2.11154		0		1.43884		0		1.27673		0		1.17731		0		0
.01	2.1105		.234545		1.43884		.283109		1.27673		.311841		1.17731		.331708		.01
.02	2.10945		.464776		1.43884		.554074		1.27673		.602826		1.17731		.633446		.02
.03	2.10835		.68651		1.43884		.801324		1.27673		.853541		1.17731		.877994		.03
.04	2.10725		.895818		1.43884		1.01441		1.27673		1.04736		1.17731		1.04338		.04
.05	1.25573		1.08914		1.43884		1.18447		1.27673		1.17155		1.17731		1.11485		.05
.06	1.25511		1.26338		1.43884		1.30462		1.27673		1.21815		1.17731		1.08619		.06
.07	1.25451		1.416		1.43884		1.37026		1.27673		1.28443		1.17731		.96022		.07
.08	1.25397		1.54505		1.43884		1.37921		1.27673		1.07298		1.17731		.748432		.08
.09	1.25341		1.64923		1.43884		1.33172		1.27673		.891506		1.17731		.469876		.09
.1	1.25289		1.7279		1.27673		1.23045		1.27673		.652147		1.17731		.149332		.1
.11	1.25241		1.78106		1.27673		1.08017		1.27673		.370594		1.17731		.184982		.11
.12	1.25193		1.8093		1.27673		.88751		1.27673		6.49459		1.17731		.303983		.12
.13	1.25148		1.81377		1.27673		.660552		1.27673		.245546		1.17731		.780296		.13
.14	1.25106		1.79612		1.27673		.408393		1.27673		.541771		1.17731		.990616		.14
.15	1.25063		1.75839		1.27673		.140682		1.27673		.809385		1.17731		1.11763		.15
.16	1.25026		1.70291		1.27673		.132847		1.27673		1.02289		1.17731		1.15137		.16
.17	1.24985		1.63227		1.27673		.40281		1.27673		1.18049		1.17731		1.08981		.17
.18	1.24939		1.54914		1.27673		.660555		1.27673		1.2707		1.17731		.93884		.18
.19	1.24891		1.45627		1.27673		.898331		1.27673		1.28961		1.17731		.711495		.19
.2	1.24842		1.35633		1.27673		1.11044		1.27673		1.23755		1.17731		.426612		.2
.21	1.24793		1.25181		1.27673		1.29148		1.27673		1.11875		1.17731		.107078		.21
.22	1.24745		1.14527		1.27673		1.4384		1.27673		.940897		1.17731		.222178		.22
.23	1.24695		1.03848		1.27673		1.54947		1.27673		.714415		1.17731		.536258		.23
.24	1.24646		.933776		1.27673		1.62444		1.27673		.541659		1.17731		.812235		.24
.25	1.24594		.832751		1.27673		1.66436		1.27673		.416059		1.17731		1.03084		.25
.26	1.24546		.736304		1.27673		1.67142		1.27673		.328735		1.17731		1.17766		.26
.27	1.24495		.64564		1.27673		1.64872		1.27673		.249644		1.17731		1.24401		.27
.28	1.24443		.561482		1.27673		1.60002		1.27673		.184856		1.17731		1.22715		.28
.29	1.24388		.484235		1.27673		1.52954		1.27673		.142454		1.17731		1.13		.29
.3	1.24331		.414307		1.27673		1.4417		1.27673		1.15999		1.17731		.960704		.3
.31	1.24273		.351542		1.27673		1.34091		1.27673		1.133646		1.17731		.731477		.31
.32	1.24215		.295832		1.27673		1.23142		1.27673		1.10749		1.17731		.50748		.32
.33	1.24156		.24695		1.27673		1.11716		1.27673		1.08113		1.17731		.287552		.33
.34	1.24095		.204444		1.27673		1.00162		1.27673		1.0548		1.17731		.07235		.34
.35	1.24034		.167864		1.27673		.887594		1.27673		1.02838		1.17731		.146431		.35
.36	1.23971		.136491		1.27673		.778196		1.27673		1.00197		1.17731		.251517		.36
.37	1.23907		.110383		1.27673		.674625		1.27673		1.07538		1.17731		1.00648		.37
.38	1.23842		.843944		1.27673		.578597		1.27673		1.04827		1.17731		1.22032		.38
.39	1.23777		.701904		1.27673		.490989		1.27673		1.02145		1.17731		1.38724		.39
.4	1.23711		.526226		1.27673		.412285		1.27673		1.2418		1.17731		1.50462		.4
.41	1.23645		.431372		1.27673		.3426		1.27673		1.12153		1.17731		1.57273		.41
.42	1.23578		.353814		1.27673		.281754		1.27673		.99845		1.17731		1.59435		.42
.43	1.23511		.296063		1.27673		.229328		1.27673		.876647		1.17731		1.57419		.43
.44	1.23443		.246866		1.27673		.184739		1.27673		.759422		1.17731		1.51838		.44
.45	1.23375		.204597		1.27673		.147289		1.27673		.649298		1.17731		1.43387		.45
.46	1.23307		.169336		1.27673		.11622		1.27673		.548052		1.17731		1.32787		.46
.47	1.23238		.140379		1.27673		.907567		1.27673		.456777		1.17731		1.20745		.47
.48	1.23169		.115953		1.27673		.701331		1.27673		.375977		1.17731		1.07915		.48
.49	1.23100		.093209		1.27673		.536317		1.27673		.305663		1.17731		.948699		.49
.5	1.23031		.072048		1.27673		.405789		1.27673		.24546		1.17731		.820867		.5
.51	1.22962		.052051		1.27673		.303757		1.27673		.194712		1.17731		.6994		.51
.52	1.22893		.033609		1.27673		.224932		1.27673		.152577		1.17731		.587018		.52
.53	1.22824		.016938		1.27673		.164746		1.27673		.118102		1.17731		.482486		.53
.54	1.22755		.002978		1.27673		.119331		1.27673		.090299		1.17731		.39573		.54
.55	1.22686		.000308		1.27673		.854672		1.27673		.061922		1.17731		.31797		.55
.56	1.22617		.000000		1.27673		.605163		1.27673		.035956		1.17731		.251873		.56
.57	1.22548		.000000		1.27673		.425553		1.27673		.021443		1.17731		.196704		.57
.58	1.22479		.000000		1.27673		.292921		1.27673		.013391		1.17731		.151454		.58
.59	1.22410		.000000		1.27673		.200153		1.27673		.005544		1.17731		.114968		.59
.6	1.22341		.000000		1.27673		.135086		1.27673		1.38553		1.17731		.860338		.6
.61	1.22272		.000000		1.27673		.900285		1.27673		.968754		1.17731		.634618		.61
.62	1.22203		.000000		1.27673		.592296		1.27673		.68244		1.17731		.461366		.62
.63	1.22134		.000000		1.27673		.364547		1.27673		.454637		1.17731		.30516		.63
.64	1.22065		.000000		1.27673		.246500		1.27673		.304981		1.17731		.233271		.64
.65	1.21996		.000000		1.27673		.155565		1.27673		.201660		1.17731		.161216		.65
.66	1.21927		.000000		1.27673		.968555		1.27673		1.31385		1.17731		.10998		.66
.67	1.21858		.000000		1.27673		.594170		1.27673		8.43112		1.17731		.747891		.67
.68	1.21789		.000000		1.27673		.358981		1.27673		5.32657		1.17731		.495891		.68
.69	1.21720		.000000		1.27673		.213490		1.27673		3.31154		1.17731		.323353		.69
.7	1.21651		.000000		1.27673		.124914		1.27673		2.02491		1.17731		.207308		.7
.71	1.21582		.000000		1.27673		.718567		1.27673		1.21711		1.17731		.130602		.71
.72	1.21513		.000000		1.27673		.406204		1.27673		7.18682		1.17731		.808038		.72
.73	1.21444		.000000		1.27673		.225426		1.27673		4.16593		1.17731		.490672		.73
.74	1.21375		.000000		1.27673		.122771		1.27673		2.36884		1.17731		.292232		.74
.75	1.21306		.000000		1.27673		.655090		1.27673		1.32036		1.17731		.170573		.75
.76	1.21237		.000000		1.27673		.343230		1.27673		7.20613		1.17731		.974910		.76
.77	1.21168		.000000		1.27673		.177752		1.27673		3.84743		1.17731		.545136		.77
.78	1.21100		.000000		1.27673		.094211		1.27673		2.00670		1.17731		.297875		.78
.79	1.21031		.000000		1.27673		.051946		1.27673		1.02265		1.17731		.158880		.79
.8	1.20962		.000000		1.27673		.028663		1.27673		1.07655		1.17731		.826163		.8
.81	1.20893		.000000		1.27673		.150956		1.27673		2.46029		1.17731		.418353		.81
.82	1.20824		.000000		1.27673		.08218		1.27673		1.16016		1.17731		.209988		.82
.83	1.20755		.000000		1.27673		.046385		1.27673		.73538		1.17731		.986385		.8

TABLE 3-6. (continued)

x	F ₈ (x)	p	F ₉ (x)	p	F ₁₀ (x)	p	F ₁₁ (x)	p	F ₁₂ (x)	p	F ₁₃ (x)	p	F ₁₄ (x)	p	F ₁₅ (x)	p	x		
0	1.07567		0		1.01216		1.01216		.961247		0		.918303		0		0		
.01	1.02018		.546325		.757431		.757431		.690636		.565738		.48094		.371998		.01		
.02	.861118		.653414		.600076		.600076		.538511		.462183		.397335		.331748		.02		
.03	.614134		.486514		.400076		.400076		.358511		.297968		.244049		.191769		.03		
.04	.30442		.101931		.15149		.15149		.137719		.103866		.0743021		.049187		.04		
.05	-.3.65144	2	1.03651		-.206229		-.206229		-.3371	2	-.855966		-.458043	2	-.762733		.05		
.06	-.374103		.937474		-.53.71		-.53.71		-.652004		.637352		-.726564		.497085		.06		
.07	-.67426		.732354		-.004.57		-.004.57		-.87202		.518371		-.893516		.142.41		.07		
.08	-.906948		.444728		-.97193		-.97193		-.965184		-.509854	2	-.911011		-.638108		.08		
.09	1.04899		.106874		-.1.01955		-.1.01955		-.91808		-.412985		-.776186		-.576996		.09		
.1	1.08655		-.243522		-.942103		-.942103		-.737681		-.711221		-.511584		-.314327		.1		
.11	1.01626		-.567666		-.749318		-.749318		-.450202		-.8994		-.161264		-.597335		.11		
.12	-.845483		-.829966		-.464.3		-.464.3		-.971627	2	-.948474		-.21664		-.840496		.12		
.13	-.591371		-.1.00194		-.123431		-.123431		-.270725		-.851013		-.559642		-.1.0253		.13		
.14	-.279		-.1.06523		-.235.32		-.235.32		-.904643		-.600919		-.622184		-.99667		.14		
.15	6.12394	2	-.1.01348		-.54174		-.54174		-.632053		-.846571		-.297214		-.930266		.15		
.16	.396719		-.852747		-.8224.00		-.8224.00		-.973123		-.074195		-.897329		-.440926		.16		
.17	.695772		-.600667		-.9956.5		-.9956.5		-.3.37318	2	-.435596		-.717969		-.733.79		.17		
.18	.9307		-.284273		1.04722		1.04722		-.818088		-.732438		-.421423		-.899774		.18		
.19	1.08029		.063067		.97707		.97707		-.558733		-.920272		5.56632	2	-.908909		.19		
.2	1.13159		.405324		.794107		.794107		.886385		.221091		.971291		-.320542		.2		
.21	1.08087		.707624		.519767		.519767		1.01548		-.148357		.678239		-.647168		.21		
.22	.933644		.939811		1.01865		1.01865		1.01865		-.499304		.655176		-.872514		.22		
.23	.703796		1.07937		-.171138		-.171138		.896125		-.784533		.335065		-.961268		.23		
.24	.412036		1.11343		.663496		.663496		-.510651		-.566214		-.3.53718	2	-.899819		.24		
.25	8.37735	2	1.03973		-.796409		-.796409		3.4944		1.02076		-.402681		-.698041		.25		
.26	-.253306		.866392		-.990093		-.990093		-.8.11771	3	-.941636		-.714494		-.387364		.26		
.27	-.57172		.610759		1.03251		1.03251		-.368811		-.739758		-.926967		1.55584	2	-.898815		.27
.28	-.846489		.297239		1.07654		1.07654		-.684976		-.441661		-.1.01079		.36087		-.703352		.28
.29	1.05707		-.4.51328	2	-.951042		-.951042		-.926501		-.8.56343	2	-.954917		.685402		-.387191		.29
.3	1.18875		-.385797		-.726719		-.726719		1.06479		.225483		-.767675		.909976		-.9.77956	3	.3
.31	1.23345		-.428855		-.428855		-.428855		1.08544		.620127		-.475062		1.00193		.371783		.31
.32	1.18991		-.948529		-.8.71905	2	-.8.71905		-.987373		.883676		-.116742		.948428		.697475		.32
.33	1.06327		1.12539		.265315		.265315		-.782362		1.04323		.259678		.757808		.316764		.33
.34	-.86416		1.21344		.395933		.395933		-.493201		1.08096		.603196		.457804		.996236		.34
.35	-.607457		1.20781		.375204		.375204		1.19715		.995683		.875844		9.10437	2	.92431		.35
.36	-.310799		1.11114		1.07554		1.07554		1.20781		.995683		1.03815		1.03815		.712414		.36
.37	6.95916	3	-.932775		1.19257		1.19257		.55291		.50136		1.07298		-.637358		.392631		.37
.38	1.27214		-.687428		1.20603		1.20603		.845769		1.52731		.972289		-.900712		1.23167	2	.38
.39	6.2899		-.393336		1.12557		1.12557		1.06207		-.215465		.763822		1.04738		-.373517		.39
.4	.909495		-.7.14673	2	-.955433		-.955433		1.18375		-.564779		.458796		1.05936		-.709442		.4
.41	1.14572		.258233		.7117.4		.7117.4		1.20217		-.690261		.936347	2	-.949402		-.949402		.41
.42	1.3339		.576403		.414461		.414461		1.11821		1.07389		-.27727		-.695015		1.03094		.42
.43	1.46999		.866528		8.55183	2	8.55183		.941364		1.18694		-.626433		-.364056		1.03094		.43
.44	1.55338		1.11561		-.25266		-.25266		.688147		1.19119		-.912924		1.03302	2	-.842087		.44
.45	1.58642		1.31469		-.579224		-.579224		.379931		1.08885		1.08851		.389835		-.380132		.45
.46	1.57384		1.45888		-.376122		-.376122		.040617		-.891475		1.19557		.739545		-.221085		.46
.47	1.5221		1.54721		1.12933		1.12933		-.305679		-.61798		1.16814		.992838		1.69907		.47
.48	1.43872		1.58209		1.32928		1.32928		-.636669		-.292188		1.03155		1.1531		.740125		.48
.49	1.53161		1.56863		1.47103		1.47103		-.933502		5.98604	2	-.800852		1.19593		.86124		.49
.5	1.20856		1.51389		1.55391		1.55391		1.18185		.412474		-.498418		1.11975		1.08331		.5
.51	1.07678		1.4261		1.58096		1.58096		1.37244		.742519		-.151036		.934782		1.18873		.51
.52	.942611		1.3137		1.55815		1.55815		1.50107		1.05108		.213113		.660963		1.16879		.52
.53	.811282		1.18574		1.49344		1.49344		1.56818		1.20452		.547232		.324879		1.02844		.53
.54	.686869		1.04943		1.39596		1.39596		1.5781		1.43464		.888175		-.043594		.743446		.54
.55	.57229		.911647		1.2751		1.2751		1.5381		1.53955		1.15804		-.414592		.452851		.55
.56	.469389		.77789		1.1399		1.1399		1.521		1.45736		1.36203		.761141	2	.9.43491		.56
.57	.379072		.65232		.993498		.993498		1.34598		1.26541		1.30357		1.06133		-.288918		.57
.58	.301476		.537814		-.357768		-.357768		1.21408		1.50176		1.57383		1.29903		-.655323		.58
.59	.236138		.436076		-.723154		-.723154		1.07107		1.4006		1.58081		1.46744		-.96047		.59
.6	.182171		.347813		-.592623		-.592623		1.25207		1.27297		1.5351		1.56272		1.24478		.6
.61	.138417		.272923		-.402747		-.402747		1.28186		1.12249		1.44157		1.58922		1.4369		.61
.62	.103578		.210706		-.338665		-.338665		.69084		.97673		1.31808		1.55508		1.55269		.62
.63	.076326		.16005		-.305232		-.305232		-.29342		.831464		1.17435		1.47145		1.7947		.63
.64	.533784	2	.119607		-.235555		-.235555		-.423015		.691018		1.02111		1.57076		1.57076		.64
.65	.3.95538	2	8.79291	2	-.178624		-.178624		-.331795		.526665		.86749		1.2064		1.49236		.65
.66	2.78046	2	6.35795	2	-.133117		-.133117		-.255473		.449044		.720467		1.04968		1.137292		.66
.67	1.92312	2	4.52083	2	-.9.74822	2	-.9.74822	2	1.193103		.351326		.583794		-.891024		1.22631		.67
.68	1.30833	2	3.16027	2	-.7.01355	2	-.7.01355	2	1.43278		.269504		.466095		-.73857		1.06556		.68
.69	8.79166	3	2.17119	2	-.4.95642	2	-.4.95642	2	1.04341		.202704		.363106		-.398198		-.901948		.69
.7	5.75367	3	1.46549	2	-.3.43743	2	-.3.43743	2	7.45636	2	1.149475		.276995		-.475628		-.744452		.7
.71	3.71601	3	9.71395	3	-.023428		-.023428		5.22726	2	1.08046		.206915		-.366674		-.59957		.71
.72	2.35644	3	6.32011	3	1.56576	4	1.56576	4	3.59371	2	7.65381	2	.151336		-.277598		-.471389		.72
.73	1.46632	3	4.03597	3	1.02626	2	1.02626	2	2.42185	2	5.31158	2	1.0835		-.205507		-.361872		.73
.74	8.94748	4	2.52435	3	6.55272	3	6.55272	3	1.59906	2	3.60962	2	7.59124	2	1.48744		-.271263		.74
.75	5.53003	4	1.54765	3	4.14833	3	4.14833	3	1.03381	2	2.40083	2	5.20247	2	1.05226		1.193337		.75
.76	5.13207	4	4.28882	4	2.55462	3	2.55462	3	6.53995	3	1.56195	3	3.48569	2	7.27281	2	1.141838		.76
.77	1.79361	4	5.45292	4	1.53873	3	1.53873	3	4.04509	3	9.33240</								

TABLE 3-6. (continued)

x	F ₁₆ (x)	p	F ₁₇ (x)	p	F ₁₈ (x)	p	F ₁₉ (x)	p	F ₂₀ (x)	p	F ₂₁ (x)	p	F ₂₂ (x)	p	F ₂₃ (x)	p	x
0	0.00000				0.00000				0.00000				0.00000				0
.01	.00000				.00000				.00000				.00000				.01
.02	.00000				.00000				.00000				.00000				.02
.03	.00000				.00000				.00000				.00000				.03
.04	.00000				.00000				.00000				.00000				.04
.05	.00000				.00000				.00000				.00000				.05
.06	.00000				.00000				.00000				.00000				.06
.07	.00000				.00000				.00000				.00000				.07
.08	.00000				.00000				.00000				.00000				.08
.09	.00000				.00000				.00000				.00000				.09
.1	.00000				.00000				.00000				.00000				.1
.11	.00000				.00000				.00000				.00000				.11
.12	.00000				.00000				.00000				.00000				.12
.13	.00000				.00000				.00000				.00000				.13
.14	.00000				.00000				.00000				.00000				.14
.15	.00000				.00000				.00000				.00000				.15
.16	.00000				.00000				.00000				.00000				.16
.17	.00000				.00000				.00000				.00000				.17
.18	.00000				.00000				.00000				.00000				.18
.19	.00000				.00000				.00000				.00000				.19
.2	.00000				.00000				.00000				.00000				.2
.21	.00000				.00000				.00000				.00000				.21
.22	.00000				.00000				.00000				.00000				.22
.23	.00000				.00000				.00000				.00000				.23
.24	.00000				.00000				.00000				.00000				.24
.25	.00000				.00000				.00000				.00000				.25
.26	.00000				.00000				.00000				.00000				.26
.27	.00000				.00000				.00000				.00000				.27
.28	.00000				.00000				.00000				.00000				.28
.29	.00000				.00000				.00000				.00000				.29
.3	.00000				.00000				.00000				.00000				.3
.31	.00000				.00000				.00000				.00000				.31
.32	.00000				.00000				.00000				.00000				.32
.33	.00000				.00000				.00000				.00000				.33
.34	.00000				.00000				.00000				.00000				.34
.35	.00000				.00000				.00000				.00000				.35
.36	.00000				.00000				.00000				.00000				.36
.37	.00000				.00000				.00000				.00000				.37
.38	.00000				.00000				.00000				.00000				.38
.39	.00000				.00000				.00000				.00000				.39
.4	.00000				.00000				.00000				.00000				.4
.41	.00000				.00000				.00000				.00000				.41
.42	.00000				.00000				.00000				.00000				.42
.43	.00000				.00000				.00000				.00000				.43
.44	.00000				.00000				.00000				.00000				.44
.45	.00000				.00000				.00000				.00000				.45
.46	.00000				.00000				.00000				.00000				.46
.47	.00000				.00000				.00000				.00000				.47
.48	.00000				.00000				.00000				.00000				.48
.49	.00000				.00000				.00000				.00000				.49
.5	.00000				.00000				.00000				.00000				.5
.51	.00000				.00000				.00000				.00000				.51
.52	.00000				.00000				.00000				.00000				.52
.53	.00000				.00000				.00000				.00000				.53
.54	.00000				.00000				.00000				.00000				.54
.55	.00000				.00000				.00000				.00000				.55
.56	.00000				.00000				.00000				.00000				.56
.57	.00000				.00000				.00000				.00000				.57
.58	.00000				.00000				.00000				.00000				.58
.59	.00000				.00000				.00000				.00000				.59
.6	.00000				.00000				.00000				.00000				.6
.61	.00000				.00000				.00000				.00000				.61
.62	.00000				.00000				.00000				.00000				.62
.63	.00000				.00000				.00000				.00000				.63
.64	.00000				.00000				.00000				.00000				.64
.65	.00000				.00000				.00000				.00000				.65
.66	.00000				.00000				.00000				.00000				.66
.67	.00000				.00000				.00000				.00000				.67
.68	.00000				.00000				.00000				.00000				.68
.69	.00000				.00000				.00000				.00000				.69
.7	.00000				.00000				.00000				.00000				.7
.71	.00000				.00000				.00000				.00000				.71
.72	.00000				.00000				.00000				.00000				.72
.73	.00000				.00000				.00000				.00000				.73
.74	.00000				.00000				.00000				.00000				.74
.75	.00000				.00000				.00000				.00000				.75
.76	.00000				.00000				.00000				.00000				.76
.77	.00000				.00000				.00000				.00000				.77
.78	.00000				.00000				.00000				.00000				.78
.79	.00000				.00000				.00000				.00000				.79
.8	.00000				.00000				.00000				.00000				.8
.81	.00000				.00000				.00000				.00000				.81
.82	.00000				.00000				.00000				.00000				.82
.83	.00000				.00000				.00000				.00000				.83
.84	.00000				.00000				.00000				.00000				.84
.85	.00000				.00000				.00000				.00000				.85
.86	.00000				.00000				.00000				.00000				.86
.87	.00000				.00000				.00000				.00000				.87
.88	.00000				.00000				.00000				.00000				.88
.89	.00000				.00000				.00000				.00000				.89
.9	.00000				.00000				.00000				.00000				.9
.91	.00000				.00000				.00000				.00000				.91
.92	.00000				.00000				.00000				.00000				.92
.93	.00000				.00000				.00000				.00000				.93
.94	.00000				.00000				.00000				.00000				.94
.95	.00000				.00000				.00000				.00000				.95
.96	.00000				.00000				.00000				.00000				.96
.97	.00000				.00000				.00000				.00000				.97
.98	.00000				.00000				.00000				.00000				.98
.99	.00000				.00000				.00000				.00000				.99

NOTE: (The exponent p is 0 unless otherwise stated.)

TABLE 3-6. (continued)

x	F ₂₄ (x)	p	F ₂₅ (x)	p	F ₂₆ (x)	p	F ₂₇ (x)	p	F ₂₈ (x)	p	F ₂₉ (x)	p	F ₃₀ (x)	p	F ₃₁ (x)	p	x
0	.762162				.736353				.710931				.685254				0
.01	.660196		.352236		.631251		.377432		.603351		.373753		.57994		.364299		.01
.02	.58155		.655373		.545913		.446115		.515135		.631673		.782838		.616254		.02
.03	.695701	4	.744324		.582937	2	.721077		.719764	2	.63094		.100633		.662808		.03
.04	.380578		.63673		.411804		.547911		.435538		.539775		.452226		.492665		.04
.05	.660361		.346734		.668079		.274061		.667584		.27167		.659811		.161294		.05
.06	.763839		.548734	2	.733966		.111063		.697857		.17251		.657095		.22315		.06
.07	.663257		.413336		.590541		.467972		.517037		.509334		.444755		.535106		.07
.08	.385315		.671274		.278493		.736624		.179571		.637822		.902528	2	.672584		.08
.09	.401524	3	.750672		.113458		.707771		.212744		.653162		.295408		.590457		.09
.1	.379061				.473805		.41442		.541476		.415546		.584566				.1
.11	.66184		.619527		.516373		.176243		.707277		.489913	2	.689798		.31523		.11
.12	.768912		.741659	2	.72745		.213332		.659851		.334614		.575478		.424222		.12
.13	.671555		.445251		.548084		.447416		.415131		.61474		.277577		.644249		.13
.14	.395342		.644327		.212302		.721256		.411172	2	.704366		.109498		.656374		.14
.15	.132998	2	.754138		.184393		.691177		.344373		.577233		.462846		.454361		.15
.16	.373347		.676552		.530223		.442231		.627314		.269737		.67007		.103524		.16
.17	.662578		.292124		.726956		.737432	2	.722349		.12141		.664961		.29223		.17
.18	.777401		.133457		.716508		.313377		.600439		.477294		.448559		.577393		.18
.19	.687272		.472237		.507048		.614073		.297615		.36455		.088913		.695593		.19
.2	.415497		.713716		.151453		.745555		9.57837	2	.684427		.300383		.571077		.2
.21	.31371	2	.701177		.248365		.64677		.442119		.471163		.59608		.270274		.21
.22	.359617		.60181		.580034		.364917	2	.691629		.111672		.704238		.1177		.22
.23	.659921		.279035		.750178		.244312		.715227		.283907		.589964		.472633		.23
.24	.788978		.122702		.710573		.403622		.325148		.540777		.288528		.673967		.24
.25	.712442		.42343		.471431		.676517		.17731		.73251		.105638		.657546		.25
.26	.450196		.737457	2	9.94559		.225277		.225277		.631031		.468591		.428015		.26
.27	.692321		.77342		.302408		.608072		.543087		.68583		.585799	2	.585799		.27
.28	.351959		.633754		.322747		.289312		.73553		.049627		.68832		.331319		.28
.29	.649821		.272232		.771726		.113744		.690765		.429696		.474261		.61746		.29
.3	.802239		.127089		.707579		.43727		.441067	2	.682707		.109793		.705732		.3
.31	.749519		.502275		.647169		.723731		.592555		.728075		.291573		.56677		.31
.32	.504425		.746384		.614803	2	.751451		.342717		.552835		.604356		.244246		.32
.33	.128757		.734167		.343951		.571464		.64651		.208325		.159206		.519206		.33
.34	.282323		.632133		.659008		.223872		.762779		.201977		.628939		.514526		.34
.35	.62545		.532201		.794373		.189359		.656066		.554339		.330595		.707484		.35
.36	.813297		.107902		.71501		.55342		.354476		.742053		.073138		.675257		.36
.37	.79826		.436725		.440419		.763812		.497227	2	.707577		.457529		.426656		.37
.38	.583317		.757331		.439768	2	.761174		.444997		.459384		.703282		.375555	2	.38
.39	.221048		.323643		.367959		.545034		.714605		.071879		.733414		.363659		.39
.4	.199463		.677466		.68471		.17449		.779702		.341295		.537113		.694823		.4
.41	.574887		.355334	2	.820917		.243283		.620353		.656514		.173224		.740626		.41
.42	.113042		.611374		.73923		.606394		.280726		.779284		.247634		.592267		.42
.43	.855288		.46633		.460078	2	.800521		.143012		.671777		.596827		.254792		.43
.44	.690498		.757657		.560869		.776191		.530419		.374377		.76696		.166837		.44
.45	.337374		.367357		.367213		.538615		.770974		.546514		.704462		.540533		.45
.46	.652316	2	.747073		.699137		.151139		.795392		.462079		.426935		.743434		.46
.47	.47741		.447436		.852683		.282032		.595125		.733761		.161822	2	.723954		.47
.48	.781977		.344613	2	.78695		.644321		.224736		.802916		.404533		.472202		.48
.49	.907204		.394377		.517424		.837684		.213388		.633784		.708446		.700333	2	.49
.5	.823203		.73213		.111693		.809216		.597754		.279584		.803132		.35433		.5
.51	.348303		.897533		.328054		.5648		.82113		.161986		.638213		.682345		.51
.52	.144242		.848767		.691077		.167352		.820977		.562163		.314545		.77966		.52
.53	.238416		.536624		.885904		.279368		.594917		.807143		.127776		.671275		.53
.54	.681011		.193985		.862937		.660385		.202779		.825715		.538855		.334432		.54
.55	.918792		.250354		.626296		.37621		.250617		.610364		.79732		.110374		.55
.56	.559254		.650237		.232316		.869231		.643688		.213325		.823606		.52317		.56
.57	.792885		.706945		.224729		.639877		.870687		.239736		.612884		.73265		.57
.58	.454294		.762165		.635711		.244265		.869719		.640021		.218123		.821628		.58
.59	.138838	2	.312437		.90281		.219397		.638817		.870116		.246857		.603402		.59
.6	.437704		.461977		.962368		.637976		.235075		.865052		.649435		.200058		.6
.61	.80874		.137696	2	.799158		.906713		.235846		.62355		.873988		.270774		.61
.62	.102055		.446337		.44703	2	.963039		.656473		.208157		.854612		.670576		.62
.63	.104397		.921568		.102214		.782905		.918353		.271841		.593587		.880617		.63
.64	.861743		.10337		.477168		.415437	2	.953993		.689732		.160604		.836675		.64
.65	.51258		.103912		.849688		.579332		.752209		.934853		.326711		.54774		.65
.66	.606004	2	.336256		.104716		.528767		.360119		.941296		.73526		.934614	2	.66
.67	.413366		.464236		.102704		.890774		.129199		.704475		.932763		.393632		.67
.68	.827171		.616150	3	.792569		.106187		.599847		.281623		.917893		.789359		.68
.69	.111179		.437943		.390388		.100349		.940977		.223407		.636287		.967052		.69
.7	.122222		.394456		.100579		.726694		.107749		.686633		.178503		.878606		.7
.71	.14315		.115531		.586181		.208238		.962613		.794654		.339008		.543901	2	.71
.72	.888983		.975945		.975945		.222777		.6337		.107177		.784399		.500051		.72
.73	.498748		.110168		.119985		.705073		.156495		.897206		.472763		.472763		.73
.74	.275609	2	.779124		.121978		.108545		.371675		.508289		.104408		.885913		.74
.75	.16355		.367095		.103392		.123669		.3941		.961273		.103079		.107324		.75
.76	.917401		.132306		.674108		.118855		.115398		.543989	3	.34569		.998939		.76
.77	.128842		.633417		.196972		.930529		.125435		.980865		.206527		.660406		.77
.78	.15474		.107754		.327869		.506771		.121238		.122919		.733029		.142956		.78
.79	.168302		.142777		.830761		.150943	2	.781107		.123849		.111703		.431153		.79
.8	.170032		.164433		.125309		.558072		.290567		.100642		.127487		.273222		.8
.81	.161702		.172575		.156486		.105		.270405		.575513	2	.17213		.123058		.81
.82	.143878		.169247		.171799		.143474		.815945		.210369		.827601		.127095		.82
.83	.123413		.150437		.174514		.167397		.128163		.566568		.305349		.103657		.83
.84	.103023		.137099		.165606		.177389		.160935		.110014		.302244		.572448		.84
.85	.809767		.11431		.148092		.173232		.177849		.151122		.89192		.336057	2	.85
.86	.60921		.903643		.125406		.158275		.179235		.175962		.138745		.673121		.86
.87	.438467		.684332		.100814		.136183		.167346		.13611		.171866				

TABLE 3-6. (continued)

x	F32(x)	p	F33(x)	p	F34(x)	p	F35(x)	p	F36(x)	p	F37(x)	p	F38(x)	p	F39(x)	p	x
0	.658437				.628866				.597612		0		.542024		0		0
.01	.548283				.519074				.487121		.529306		.440565		.305112		.01
.02	.25464				.227994				.201835		.537299		.444425		.494495		.02
.03	-.124334				-.142829				-.153011		.547186		.457735		.426273		.03
.04	-.461928				-.463998				-.455913		.555331		.430634		.303679		.04
.05	-.645208				-.623375				-.591240		.522764	2	.545524		.537384	3	.05
.06	-.612758				-.565196				-.5290824		.503007		.491562		.501274		.06
.07	-.575224				-.509549				-.468010		.526932		.471315		.433507		.07
.08	-.518018				-.456774				-.403471		.536803		.440417		.446267		.08
.09	.35616				.400213				.433492		.581325		.493566		.515032		.09
.1	.605658				.606723				.613676		.687903		.683637		.683502	2	.1
.11	.653101				.601787				.529592		.527183	4	.482466		.507104		.11
.12	.482264				.586717				.514651		.516093		.483636		.483636		.12
.13	.149782				.561056	2			.461735		.566189		.414213		.414213		.13
.14	-.233647				.528047				.500472		.407532		.411194		.411194		.14
.15	-.340115				.57876				.505139		.980681	2	.545572		.635950	2	.15
.16	-.667124				.628293				.620003		.248329		.472711		.225613		.16
.17	-.571804				.458795				.476249		.504578		.427096		.496321		.17
.18	-.285296				.128673				.619233		.575374	3	.107496		.509103		.18
.19	.974737	2			.247554				.543568		.552231		.434168		.523433		.19
.2	.449265				.538967				.573518		.511111		.512234		.545564		.2
.21	.652794				.643755				.685921		.685921		.685921		.685921		.21
.22	.639758				.524704				.431287		.8113	2	.92222		.46131		.22
.23	.413655				.222335				.615849		.2119		.584409		.905132	2	.23
.24	.487891	2			.158933				.582411		.511682		.461201		.354588		.24
.25	-.334166				.484648				.34184		.580600		.167763		.552563	2	.25
.26	-.608147				.647153				.605354		.605354		.504331		.292356		.26
.27	-.681721				.563311				.379188		.43065		.266439		.503879		.27
.28	-.529514				.315358				.604392		.131047		.595237		.524501		.28
.29	-.200803				.621743	2			.616475		.757409		.48975		.339905		.29
.3	.19654				.421512				.409926		.541148		.205047		.557118		.3
.31	.531485				.656945				.656945		.621400		.520822		.291925		.31
.32	.693008				.632929				.47713		.483891		.288714		.505805		.32
.33	.62762				.409468				.584603		.155316		.601931		.532338		.33
.34	.353181				.424735				.644953		.212472		.519241		.357157		.34
.35	-.389584	2			.342083				.477436		.520213		.244649		.641756		.35
.36	-.42144				.611396				.139674		.636417		.122247		.292075		.36
.37	-.668526				.671601				.250323		.237224		.447183		.517507		.37
.38	-.698172				.500334				.555226		.22096		.610292		.547875		.38
.39	-.499062				.154837				.666748		.104274		.550227		.369829		.39
.4	-.134611				.247268				.543945		.49384		.287126		.569874		.4
.41	.277397				.568047				.228074		.642121	2	.55915		.292852		.41
.42	.602932				.696483				.171345		.571021		.428148		.327047		.42
.43	.739239				.586728				.514521		.287035		.618266		.562894		.43
.44	.629625				.274237				.680245		.133309		.381336		.384697		.44
.45	.318242				.135821				.608541		.460761		.333324		.579823		.45
.46	-.099935				.504019				.322161		.656953		.041782		.294546		.46
.47	-.490722				.365775				.080719		.61864		.409956		.550001		.47
.48	-.727772				.467174	2			.460206		.358017		.625523		.581047		.48
.49	-.735219				.399698				.68321		.544243	2	.61829		.402201		.49
.5	-.502903				.614001	3			.669946		.419567		.537789		.677525	2	.5
.51	.108088				.415374				.422218		.658677		.603313		.256929		.51
.52	.326871				.589119				.235485	2	.466791		.579661		.523358		.52
.53	.663801				.732929				.339104		.435305	2	.63176		.602772		.53
.54	.794472				.306625				.672333		.447534		.656279		.422839		.54
.55	.67499				.143077				.725971		.3618		.440984		.788719	2	.55
.56	.339984				.207393				.526123		.655635		.609487		.300667		.56
.57	-.107641				.644989				.143038		.714416		.347638		.571786		.57
.58	-.529491				.332402				.296937		.319683		.636186		.629414		.58
.59	-.792836				.658692				.642836		.136567		.697541		.447932		.59
.6	-.613693				.312968				.773314		.30215		.50349		.612404		.6
.61	-.582115				.143824				.235465		.641987		.125468		.676175	2	.61
.62	-.165833				.561778				.283189		.760971		.307762		.594233		.62
.63	.310111				.601819				.177633		.612028		.637812		.652074		.63
.64	.701219				.781587				.581268		.245277		.742872		.478531		.64
.65	.087264				.36274				.809475		.217122		.580293		.108192		.65
.66	.808641				.526721	2			.504335		.613597		.203331		.311537		.66
.67	.484437				.42573				.440625		.603699		.256324		.622879		.67
.68	.711736	3			.777118				.225171	2	.716381		.634479		.703514		.68
.69	-.484767				.08508				.491341		.375648		.792766		.517384		.69
.7	-.846923				.709627	2			.812702		.10522		.668841		.128542		.7
.71	-.971362				.300771				.867011		.563131		.300053		.772432		.71
.72	-.817536				.216434				.632996		.638602		.187636		.659591		.72
.73	-.423887				.680602				.181005		.631683		.622717		.756903		.73
.74	.103153				.944256				.345497		.53969		.848061		.567662		.74
.75	.619004				.919822				.775699		.473205	2	.776996		.154818		.75
.76	.981185				.607488				.96477		.475777		.425426		.331761		.76
.77	.108788				.953437	2			.84893		.855325		.909033	2	.709181		.77
.78	.904954				.466123				.431505		.256318		.307769		.82952		.78
.79	.474071				.135606				.107676		.740443		.908222		.636877		.79
.8	-.931355	2			.105802				.656785		.263272		.913515		.692399		.8
.81	-.474066				.266151				.102314		.32413		.596642		.347072		.81
.82	-.110935				.108644				.632261		.545518	2	.534283		.778724		.82
.83	-.12988				.403767	2			.815678		.109136		.534726		.930473		.83
.84	-.119414				.638717				.280173		.133665		.970224		.737949		.84
.85	-.813235				.111236				.372198		.58956		.109515		.247062		.85
.86	-.231695				.133125				.955913		.439018	2	.850822		.370088		.86
.87	.438406				.118522				.130157		.777079		.29811		.887021		.87
.88	.107615				.725937				.130253		.119916		.597059		.110044		.88
.89	.157847				.493533	2			.94293		.13626		.101585		.906398		.89
.9	.187948				.297098				.13047		.135316		.745647		.344448		.9
.91	.195903				.190732				.543069		.128126		.128126		.406933		.91
.92	.183974				.125024				.265659		.788975		.137266		.108461		.92
.93	.157512				.070622				.182891		.110533	2	.152607		.142654		.93
.94	.123306				.09214				.212836		.179726		.93658		.126137		.94
.95	.879364				.177777				.212301		.219427		.175174		.570725		.95
.96	.565112				.125336				.185746		.224297		.169406		.497451		.96
.97	.320																

TABLE 3-6. (continued)

x	F ₄₀ (x)	p	F ₄₁ (x)	p	F ₄₂ (x)	p	F ₄₃ (x)	p	F ₄₄ (x)	p	F ₄₅ (x)	p	x
0	.526571		0		.568037		0		.584876		0		0
.01	.422243		.329513		.456988		.345492		.46527		.362593		.01
.02	.149261		.530441		.167686		.553331		.155213		.574751		.02
.03	-.187691		.525022		-.185627		.557923		-.218872		.548391		.03
.04	-.459356		.314992		-.463384		.307328		-.504395		.294321		.04
.05	-.56144		-.181236	2	-.555768		-.470913	2	-.584782		-.841866	2	.05
.06	-.454615		-.34485		-.426092		-.383347		-.427031		-.424999		.06
.07	-.179661		-.538064		-.125359		-.565891		-.525556	2	-.591643		.07
.08	.158088		-.522435		.227777		-.521162		.275335		-.512776		.08
.09	.429049		-.503795		.493716		-.266771		.533474		-.220725		.09
.1	.529059		3.29995	2	.567297		9.54944	2	.573527		.163562		.1
.11	.419372		.35694		.418886		.420312		.378687		.480569		.11
.12	.141631		.541748		.10677		.57708		2.80379	2	.598362		.12
.13	-.197846		.515026		-.24395		.502394		-.355492		.467459		.13
.14	-.468883		.286568		-.499793		.22569		-.563318		.141621		.14
.15	-.567165		-.552318	2	-.554		-.142314		-.56185		-.244162		.15
.16	-.454434		-.377568		-.386491		-.453347		-.531045		-.52952		.16
.17	-.173536		-.554796		-.628615	2	-.583348		3.55043	2	-.595302		.17
.18	.168601		-.517386		.28927		-.479027		.38822		-.41331		.18
.19	.440201		-.279211		.530522		-.179374		.582624		-.584203	2	.19
.2	.536742		6.75475	2	.56487		.193143		.538441		.322138		.2
.21	.420369		.387993		.377843		.489799		.272744		.569763		.21
.22	.134878		.556906		4.26155	2	.590738		-.106564		.580517		.22
.23	-.210743		.507595		-.30856		.454568		-.444481		.548753		.23
.24	-.483888		.258174		-.536455		.154523		-.602167		-.299802	2	.24
.25	-.579161		-.095078		-.549999		-.240711		-.513948		-.538217		.25
.26	-.459014		-.414851		-.342649		-.520191		-.214687		-.601945		.26
.27	-.165226		-.576144		4.54003	3	-.590518		.173894		-.554972		.27
.28	.181455		-.515104		.354526		-.422011		.492613		-.27542		.28
.29	.456547		-.254317		.568303		-.8.12501	2	.609947		.121113		.29
.3	.550345		.105471		.560056		.295227		.476279		.469211		.3
.31	.425512		.424166		.331589		.555391		.144906		.622535		.31
.32	.128582		.576771		-.2.77569	2	.593206		-.249086		.515307		.32
.33	-.227359		.502314		-.375933		.391754		-.543761		.190581		.33
.34	-.505699		.228147		-.274313		3.08202	2	.616803		-.216872		.34
.35	-.598674		-.140065		-.542664		-.544578		-.43628		-.536334		.35
.36	-.469055		-.45902		-.291719		-.582327		-.7.46308	2	-.632564		.36
.37	-.165028		-.6035		8.03924	2	-.584766		.320235		-.463045		.37
.38	.197643		-.515503		.426396		-.548779		.585278		-.9.68451	2	.38
.39	.479592		-.227534		.608063		.032011		.609558		.513308		.39
.4	.571298		.149271		.551376		.404252		.380574		.594524		.4
.41	.435522		.467902		.276664		.616496		-.9.01935	3	.626523		.41
.42	.122334		.602678		-.108499		.580752		-.399476		.593118		.42
.43	-.249309		.478704		-.451285		.50898		-.628516		-.9.67444	3	.43
.44	-.536687		.194138		-.614112		-.9.05023	2	-.59911		-.412841		.44
.45	-.628048		-.19374		-.529772		-.456411		-.320514		-.645091		.45
.46	-.48601		-.51361		-.22914		-.63866		9.41772	2	-.60555		.46
.47	-.162808		-.639068		.169951		-.560602		.473684		-.507646		.47
.48	.218796		-.518538		.50874		-.250837		.659086		.124911		.48
.49	.511989		-.196471		.650551		.166897		.570294		.509163		.49
.5	.602223		.202855		.535866		.522928		.740695		.680053		.5
.51	.451307		.525078		.207283		.670404		-.195016		.561272		.51
.52	.11.476		.636767		-.206057		.545798		-.555572		.199445		.52
.53	-.273454		.49596		-.554992		.196305		-.688325		-.25368		.53
.54	-.581184		.192233		-.253133		-.23768		-.553827		-.604469		.54
.55	-.671662		-.262054		-.5068		-.578642		-.152512		-.68991		.55
.56	-.512684		-.584386		-.5068		-.684315		.296479		-.49437		.56
.57	-.161904		-.684537		-.281755		-.506941		.629659		-.7.15034	2	.57
.58	.247861		-.524043		.607316		-.11479		.698558		.58918		.58
.59	.558676		-.156852		.659869		.533706		.470989		.688023		.59
.6	.648079		.273156		.507263		.6536		3.79098	2	.691515		.6
.61	.477063		.596228		.112695		.709462		-.420361		.39202		.61
.62	-.106725		.68265		-.551616		.472662		-.708704		-.8.57476	2	.62
.63	-.523415		.492218		-.645881		3.57629	2	-.700051		-.55588		.63
.64	-.647787		9.47611	2	-.699157		-.423068		-.591394		-.759441		.64
.65	-.738314		-.554669		-.463721		-.712172		9.16045	2	-.65221		.65
.66	-.554789		-.685316		-.2.89049	2	-.706105		.544975		-.253465		.66
.67	-.162434		-.752813		.432284		-.599951		.771942		.267015		.67
.68	.290883		-.531125		.731513		8.51703	2	.668051		.68019		.68
.69	.629912		-.9.59598	2	.741665		.549256		.269292		.797875		.69
.7	.719239		.374459		.450673		.795222		-.258869		.558373		.7
.71	.516828	2	.701982		-.3.00182	2	.712245		-.689055		.588266	2	.71
.72	9.28912		.746875		-.507294		.325633		-.829074		-.482479		.72
.73	-.394116		.481987		-.763135		-.209043		-.607544		-.82012		.73
.74	-.756337		3.49788	3	-.736141		-.667517		-.11016		-.792107		.74
.75	-.848505		-.503372		-.374541		-.849025		.45156		-.597749		.75
.76	-.625745		-.835405		.162202		-.665131		.828595		.194397		.76
.77	-.16446		-.852258		.655989		-.180921		.843761		.716062		.77
.78	.362969		-.555349		.897586		.405045		.47474		.918908		.78
.79	.751925		-.4.03858	4	.774454		.840546		-.127685		.692094		.79
.8	.84193		.54328		.522587		.926833		-.698325		.122871		.8
.81	.284476	2	.873023		-.281044		.608662		-.271643		-.532254		.81
.82	.632564		.842178		-.185531		7.11843	3	-.804762		-.599325		.82
.83	-.530488		-.445576		-.968086		-.622845		-.25032		-.934613		.83
.84	-.966945		-.179775		-.730618		-.996991		.451899		-.441777		.84
.85	.1.06549		-.783945		-.150017		-.927243		.974144		.302424		.85
.86	-.767432		-.1.11717		.542283		-.414481		1.04913		.935186		.86
.87	-.166245		-.1.02236		1.04829		.335529		.604671		1.1176		.87
.88	.518743		-.50863		1.12738		.987561		-.180766		.712834		.88
.89	1.01838		.238734		.707114		1.21961		-.93777		-.11652		.89
.9	1.11371		.918811		-.6.74407	2	.876301		-.1.26779		-.954303		.9
.91	.724068		1.22771		-.873061		6.49872	2	-.943813		-.1.32052		.91
.92	-.4.99329	2	.98367		-.1.51021		-.857528		-.5.79512	2	-.932312		.92
.93	-.947223		.216991		-.1.16649		-.1.41891		.967402		-.088879		.93
.94	-.1.62637		-.815171		-.359375		-.1.24672		1.54047		1.19156		.94
.95	-.1.78163		-.1.68822		.802808		-.284867		1.79629		1.61969		.95
.96	-.1.25132		-.1.56771		1.77771		1.08268		-.077998		-.851882		.96
.97	-.6.65468	2	-.1.59031		1.95759		2.07231		-.1.63192		-.881422		.97
.98	1.45122		-.194502		.799922		1.7206		-.2.18005		-.2.20709		.98
.99	2.96251		2.40063		-.1.73892		-.769126		-.122665		-.939016		.99
1	.4.631		4.72416		-.5.3888		-.5.6654		.5.89781		6.01841		1

NOTE: (The exponent p is 0 unless otherwise stated.)

Table 3-7. Eigenvalues for $c = 25\pi$ or aperture length = 25λ

n	λ_n	n	λ_n
0	.28284	28	.28284
1	j.28284	29	j.28284
2	-.28284	30	-.28284
3	-j.28284	31	-j.28284
4	.28284	32	.28284
5	j.28284	33	j.28284
6	-.28284	34	-.28284
7	-j.28284	35	-j.28284
8	.28284	36	.28284
9	j.28284	37	j.28284
10	-.28284	38	-.28284
11	-j.28284	39	-j.28284
12	.28284	40	.28284
13	j.28284	41	j.28283
14	-.28284	42	-.28281
15	-j.28284	43	-j.28277
16	.28284	44	.28267
17	j.28284	45	j.28239
18	-.28284	46	-.28144
19	-j.28284	47	-j.27768
20	.28284	48	.26430
21	j.28284	49	j.22848
22	-.28284	50	-.16622
23	-j.28284	51	-j.10076
24	.28284	52	.05421
25	j.28284	53	j.02320
26	-.28284	54	-.00908
27	-j.28284	55	-j.00368

Table 3-8. Eigenfunctions for $c = 25\pi$ or aperture length = 25λ

$$\psi_n(x) = F_n(x) \times 10^p$$

x	F ₀ (x)	p	F ₁ (x)	p	F ₂ (x)	p	F ₃ (x)	p	F ₄ (x)	p	F ₅ (x)	p	F ₆ (x)	p	F ₇ (x)	p	x
0	2.23337		0		1.5715		0		1.35405		0		1.22952		0		0
.01	2.2247		.277483		1.54104		.335313		1.3072		.369314		1.16863		.393947		.01
.02	2.19889		.548569		1.45094		.632594		1.16997		.708591		.99203		.743178		.02
.03	2.15653		.807102		1.30488		.934886		.951976		.988		.71728		1.00815		.03
.04	2.09857		1.0474		1.10881		1.16729		.66842		1.13486		.371613		1.15901		.04
.05	2.02631		1.26445		.870602		1.33794		.338932		1.28309		1.08737		1.17901		.05
.06	1.94132		1.4541		.59962		1.4381		1.39196	2	1.27504		1.392714		1.0663		.06
.07	1.84541		1.61317		.306271		1.46363		1.36229		1.16199		1.736855		.833952		.07
.08	1.74054		1.73951		1.45579	3	1.41406		1.69435		.953879		1.01033		.508334		.08
.09	1.62879		1.83209		1.503949		1.293		1.97825		.668253		1.18745		.125882		.09
.1	1.51226		1.89092		1.599599		1.10765		1.19883		.328616		1.25212		1.271151		.1
.11	1.39302		1.91703		1.876124		.868217		1.34341		1.77219	2	1.19923		.639527		.11
.12	1.27307		1.91233		1.1255		.587157		1.40421		1.402013		1.03477		.939817		.12
.13	1.15423		1.87949		1.34133		1.27356		1.37901		1.736472		1.774956		1.14062		.13
.14	1.03816		1.82179		1.519		1.37119	2	1.27103		1.01646		1.444313		1.22174		.14
.15	.926321		1.74293		1.65574		1.364922		1.08846		1.22231		1.30932	2	1.17994		.15
.16	.819903		1.64684		1.74293		1.672157		1.84341		1.34058		1.305763		1.0094		.16
.17	.719867		1.53756		1.54684		1.953986		1.551049		1.16474		.699735		.740309		.17
.18	.626921		1.4191		1.35756		1.20113		1.22827		1.29526		.99607		.396695		.18
.19	.541533		1.29515		1.17952		1.40674		1.07502		1.13907		1.18186		1.30336	2	.19
.2	.463948		1.16917		1.174675		1.56655		.43946		1.908575		1.31034		.373521		.2
.21	.394206		1.0442		1.66887		1.67873		.75241		1.620305		1.33722		.726519		.21
.22	.332174		.92281		1.57019		1.74399		1.03555		1.293381		1.26296		1.01574		.22
.23	.275568		.807108		1.45609		1.76475		1.27302		1.20949	2	1.09574		1.21573		.23
.24	.229991		.598599		1.33176		1.74242		1.46412		.396549		.850099		1.31171		.24
.25	.188957		.598728		1.20197		1.69085		1.60359		.722347		.545215		1.29829		.25
.26	.15392		.507901		1.071		1.60773		1.69041		1.01426		.20298		1.17953		.26
.27	.124302		.426543		1.942475		1.30235		1.72738		1.26125		1.35983		.967838		.27
.28	.935129	2	.354645		1.819331		1.38117		1.69041		1.71867		1.504124		.682063		.28
.29	.789708	2	.291933		1.703824		1.25027		1.67027		1.59389		1.828462		.345311		.29
.3	.662116		.23792		1.597541		1.11516		1.58919		1.67573		1.11169		1.74659	2	.3
.31	.484232	2	.191971		1.501462		1.98058		1.48289		1.70453		1.3428		1.581729		.31
.32	.374091	2	.153352		1.416035		1.850425		1.35884		1.68494		1.51537		1.72594		.32
.33	.286373	2	.121277		1.341257		1.727697		1.22406		1.62729		1.62729		1.02983		.33
.34	.217209	2	.94981	2	1.26775		1.614547		1.08487		1.55191		1.68037		1.28176		.34
.35	1.63218	2	.735852		1.221964		1.312331		.946607		1.41487		1.67955		1.47271		.35
.36	1.21493	2	.64502	2	1.17602		1.421717		.813614		1.28139		1.63211		1.59937		.36
.37	.895746	3	.426629	2	1.138028		1.342791		.869141		1.14055		1.54679		1.66296		.37
.38	.654051	3	.522109	2	1.107028		1.275185		.575428		.997434		1.43293		1.66847		.38
.39	.472909	3	.295949	2	1.20613	2	1.218194		.473791		.857943		1.29981		1.62366		.39
.4	.38551	3	1.76284	2	1.28554	2	1.170886		.584763		.726238		1.15604		1.53801		.4
.41	.29934	3	1.28554	2	1.046632	2	1.1322		.308238		.605256		1.00913		1.42178		.41
.42	1.68313	3	9.24568	3	1.35569	2	1.01022		.241624		.496815		1.86528		1.28513		.42
.43	1.16851	3	6.25783	3	1.25519	2	1.762534		.189993		.40176		.72927		1.13748		.43
.44	.802730	4	4.44265	3	1.83323	2	1.568516		1.16206		.32015		.604456		.95599		.44
.45	.545571	4	2.22948	3	1.31209	2	1.18643	2	1.11024		.251436		.492306		1.40318		.45
.46	.366777	4	2.22948	3	1.31209	2	1.18643	2	1.11024		.251436		.492306		1.40318		.46
.47	1.43860	4	1.51872	3	1.48633	3	1.218662	2	6.15152	2	1.48535		1.32512		1.576963		.47
.48	1.60318	4	1.02235	3	1.47888	3	1.55066	2	4.48818	2	1.11742		1.243087		1.465754		.48
.49	1.04191	4	6.80360	4	1.05505	3	1.010857		.032309		8.28706		1.156197		1.369669		.49
.5	.649250	5	4.47242	4	1.205812	3	1.750409	3	.022946		6.05869	2	1.140455		1.288538		.5
.51	4.24777	5	2.90399	4	1.36913	3	1.511929	3	1.60757	2	4.36646	2	1.104347		1.221565		.51
.52	2.66349	5	1.86194	4	1.899187	4	1.344644	3	1.11083	2	3.10185	2	1.763483	2	1.67368		.52
.53	1.64939	5	1.17832	4	1.82903	4	1.26928	3	1.57007	3	2.17175	2	1.50157	2	1.2439		.53
.54	1.30837	5	7.36060	5	1.37289	4	1.50005	3	1.07620	3	1.49844	2	1.50405	2	1.090961		.54
.55	.608475	6	4.53391	5	1.35338	4	1.50005	3	1.07620	3	1.49844	2	1.50405	2	1.090961		.55
.56	3.62310	6	2.75247	5	1.46494	4	1.50005	3	1.07620	3	1.49844	2	1.50405	2	1.090961		.56
.57	2.12716	6	1.64529	5	1.89141	5	1.387947	4	1.41570	3	4.50066	3	1.27109	2	1.22601	2	.57
.58	1.23166	6	9.66488	6	1.53972	5	1.40116	4	8.96891	4	2.92356	3	1.47296	3	1.20976		.58
.59	7.01543	7	5.55839	6	1.24312	5	1.46411	4	1.59792	4	1.86961	3	1.55779	3	1.48864	2	.59
.6	3.93055	7	3.12948	6	1.90457	5	1.79183	5	3.43989	4	1.17673	3	1.38651	3	1.86099	3	.6
.61	2.16487	7	1.69988	6	1.10136	5	1.19726	5	2.08038	4	1.28699	4	1.27628	3	1.42144	3	.61
.62	1.16873	7	8.75939	7	1.626897	6	1.02541	5	1.23784	4	4.43836	4	1.42046	3	1.10972	3	.62
.63			4.08999	7	1.31046	6	1.172997	5	1.24330	5	2.65789	4	1.71234	4	1.58420	3	.63
.64			1.48484	7	1.93350	6	1.93438	6	4.16649	5	1.56432	4	1.52023	4	1.59597	3	.64
.65				7	1.04859	6	1.58237	6	2.35459	5	9.04425	5	1.10725	4	1.67712	4	.65
.66				7	1.57949	6	1.92348	6	1.30671	5	5.13436	5	1.80524	4	1.75840	4	.66
.67				7	1.57770	6	1.11750	6	7.11750	6	2.86048	5	1.02904	4	1.36114	4	.67
.68				7	1.48673	7	1.15785	7	5.80149	6	1.56302	5	1.75211	5	1.92344	4	.68
.69				7	1.33850	7	1.19044	7	1.39014	6	8.37141	6	1.15116	5	1.07852	4	.69
.7																	.7
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TABLE 3-8. (continued)

x	F ₈ (x)	p	F ₉ (x)	p	F ₁₀ (x)	p	F ₁₁ (x)	p	F ₁₂ (x)	p	F ₁₃ (x)	p	F ₁₄ (x)	p	F ₁₅ (x)	p	x
0	1.14377		0		1.07882		0		1.02667		0		.983074		0		0
.01	1.07038		.411991		.99411		.425933		.931608		.436867		.8785		.445463		.01
.02	.85964		.765267		.753281		.779051		.66402		.74688		.587019		.790219		.02
.03	.518632		1.00956		.394153		.99906		.273431		.980515		.17059		.956566		.03
.04	.148501		1.11026		2.69696	2	1.04851		.16796		.979401		.282352		.90437		.04
.05	-.260921		1.05329		-.444238		.919157		-.578725		.76389		.67576		.651783		.05
.06	-.637597		.847026		-.792649		.635217		-.883303		.432747		.72264		.249882		.06
.07	-.935985		.520904		-.101822		.239421		-.102592		-.423306	3	-.981439		-.208786		.07
.08	-.111307		.121047		-.108634		-.195481		-.980716		-.441137		.829471		-.62124		.08
.09	-.115291		-.296443		-.986948		-.598186		-.755288		-.792053		-.502685		-.895177		.09
.1	-.104923		-.673506		-.735801		-.901278		-.393798		-.988442		-.678022	2	-.969582		.1
.11	-.815582		-.958278		-.371742		-.10545		4.08292	2	-.992307		.37861		-.82813		.11
.12	-.481347		-.111219		.049612		-.103291		.469135		-.803311		.749238		-.50248		.12
.13	-.877335	2	-.111302		.464596		-.840721		.814422		-.458432		.965458		-.648529	2	.13
.14	.317564		-.967303		.811245		-.510088		1.01548		-.243602	2	.983018		.388253		.14
.15	.686334		-.68976		1.03864		-.933795	2	1.03727		.415702		.798778		.757704		.15
.16	.975657		-.319998		1.11406		-.336109		.876661		.778178		.450828		.963065		.16
.17	1.13283		9.29566	2	1.02757		.715297		1.52446		.994992		1.01559	2	.960239		.17
.18	1.19886		.495462		.792725		.982424		.149665		1.02608		-.434218		.750332		.18
.19	1.11011		.836391		.444061		1.09631		-.290452		.866473		-.79343		.378808		.19
.2	.898103		1.07361		3.14601	2	1.04034		-.683071		.546403		-.99651		-.074904		.2
.21	.587528		1.179		-.587368		.82435		-.96254		.125052		1.0041		-.514814		.21
.22	.212664		1.14135		-.755206		.482316		1.08318		-.32092		-.815555		-.84887		.22
.23	-.185889		.96705		-.102318		6.64041	2	-.102632		-.711642		-.46821		-.10081		.23
.24	-.367806		.678435		-.15705		-.361487		-.802521		-.978387		-.293204	2	-.960443		.24
.25	-.895404		.310317		-.141112		-.739241		-.448945		1.07559		.417532		-.716481		.25
.26	-.113828		-.947745	2	-.979459		-.101359		-.244535	2	-.987167		.788661		-.326349		.26
.27	-.127569		-.491941		-.694364		-.14737		.4101		-.730161		1.01592		.131359		.27
.28	-.129786		-.839122		-.322556		-.322556		.782669		-.348887		1.05879		.565967		.28
.29	-.120605		-.110154		9.02222	2	-.9489		1.04011		9.25933	2	.910537		.892906		.29
.3	-.101148		-.125495		.495344		-.647336		1.14598		.522029		.600104		1.04996		.3
.31	-.733275		-.128731		.847297		-.260158		1.08705		.871078		.182001		1.00852		.31
.32	-.395994	2	-.119906		1.10878		1.6261		.874001		1.08607		-.270536		.777804		.32
.33	-.267822		-.100188		1.25433		.56876		.53858		1.13574		-.680782		.401835		.33
.34	.347304		-.716458		1.27219		.910604		.127898		1.0149		-.981268		-.050047		.34
.35	.701634		-.369507		1.16456		1.15047		-.302991		.743825		-.112455		-.496753		.35
.36	1.01958		9.45383	3	.945081		1.26442		.699		.563691	2	-.108993		-.860392		.36
.37	1.27473		.391115		.638106		1.24385		-.101221		-.06631		-.885317		-.107973		.37
.38	1.46922		.749206		-.273156		1.09494		-.120742		-.499467		-.544321		-.111987		.38
.39	1.59574		1.06229		-.117669		.836704		-.126542		-.86665		-.119511		-.976682		.39
.4	1.65587		1.31506		-.503583		.497285		-.118388		-.112713		.326725		-.675731		.4
.41	1.65535		1.49874		-.856932		.110031		-.97594		1.25163		.732006		-.266325		.41
.42	1.60288		.61082		-.1572		-.290645		-.667025		1.22998		1.04785		.187816		.42
.43	1.50898		1.63426		-.139008		-.67293		-.290599		1.06862		1.22136		.619376		.43
.44	1.38487		1.63627		-.134869		-.101045		.116529		-.789928		1.24897		.967793		.44
.45	1.24149		1.56703		-.163279		-.128397		.51861		-.42682		1.12698		1.18754		.45
.46	1.08878		1.49832		-.164767		-.148229		.884589		-.178257	2	.874328		1.25323		.46
.47	.935223		1.32225		-.160274		-.140188		1.19063		.39802		.523243		1.16119		.47
.48	.787517		1.17033		-.150996		-.164591		1.42141		.785583		.113665		.927692		.48
.49	.650561		1.01269		-.138234		-.162284		1.57027		1.11688		-.312495		.584537		.49
.5	.527525		.857659		-.123271		-.154458		1.63835		1.57296		-.716517		-.173141		.5
.51	.420063		.711591		-.107272		-.142487		1.63314		1.94436		-.106681		-.261894		.51
.52	.328587		.578695		-.912193		-.127769		1.5665		1.63051		-.134132		-.678644		.52
.53	.252557		.461613		-.758782		-.111605		1.45282		1.63809		-.132834		-.104244		.53
.54	.190773		.617917		-.931122		-.931122		1.30712		1.57909		-.1626		-.132878		.54
.55	.141635		.277596		-.492944		-.791798		1.14565		1.68855		-.164062		-.132436		.55
.56	.10356		.209326		-.385413		-.644494		.974823		1.32254		-.58453		-.162664		.56
.57	.741403	2	1.15504		-.295441		-.513282		.81059		1.15644		-.147361		-.16421		.57
.58	.522714	2	1.12706		-.222095		-.400185		.658208		.963798		-.132499		-.138378		.58
.59	.362194	2	8.04706	2	-.163757		-.305561		.522335		.815404		-.115516		-.146863		.59
.6	2.46621	2	5.64152	2	-.118438		-.228555		.40533		.659186		-.978522		-.131492		.6
.61	1.64988	2	3.88313	2	-.084027		-.1675		.307699		.520199		-.806631		-.14012		.61
.62	1.08422	2	2.62381	2	-.584739	2	-.120284		.228573		.400977		-.647814		-.959409		.62
.63	.699798	3	1.74005	2	-.399091	2	-.846388	2	.116811		.302028		-.507301		-.784776		.63
.64	.443318	3	1.13231	2	-.583542	2	-.583542	2	.118257		.222369		-.387603		-.624715		.64
.65	.275657	3	7.22803	3	-.179237	2	-.394145	2	8.23673	2	.160055		-.289067		-.484372		.65
.66	1.68154	3	4.52454	3	-.112707	2	-.260756	2	5.61468	2	.112631		-.210482		-.366013		.66
.67	1.00587	3	2.77626	3	-.710152	3	-.168924	3	3.74507	2	7.74846	2	-.149656		-.269633		.67
.68	.589743	4	1.66910	3	-.438239	3	-.107122	2	2.44372	2	5.21053	2	-.103905		-.193754		.68
.69	.358719	4	9.82708	4	-.24745	3	-.664707	3	.015594		5.42419	2	-.704375	2	-.135746		.69
.7	1.90461	4	5.66293	4	-.156487	3	-.403402	3	9.72760	3	2.19838	2	-.466124	2	-.927585	2	.7
.71	1.04781	4	3.19197	4	-.904497	3	-.239316	3	5.92907	3	1.37831	2	-.301023	2	-.618022	2	.71
.72	.53562	5	1.75861	4	-.510886	4	-.138695	3	3.52904	3	8.45474	3	-.189636	2	-.401377	2	.72
.73	.296093	5	9.46312	5	-.281781	4	-.784707	4	2.04992	3	5.03532	3	-.116479	2	-.253994	2	.73
.74	1.51833	5	4.96893	5	-.151637	4	-.433079	4	1.16120	3	2.93033	3	-.697130	3	-.156528	2	.74
.75	.739222	5	2.54324	5	-.795408	5	-.232944	4	6.40908	4	1.66112	3	-.406263	3	-.938809	3	.75
.76	.349855	6	1.26745	5	-.406266	5	-.121992	4	3.44343	4	9.16403	4	-.230337	3	-.347582	3	.76
.77	1.75604	6	6.14500	6	-.201814	5	-.621330	5	1.78964	4	4.91307	4	-.126930	3	-.310325	3	.77
.78	8.11098	7	2.88830	6	-.973697	6	-.307396	5	9.12758	5	2.55994	4	-.679091	4	-.170697	3	.78
.79	.562681	7	1.31452	6	-.455580	6	-.147533	5	4.49133	5	1.29301	4	-.532288	4	-.910224	4	.79
.8	1.59418	7	5.74216	7	-.206161	6	-.685825	6	2.13981	5	6.32396	5	-.176944	4	-.469869	4	.8
.81	—	—	2.40069	—	-.898361	7	-.308209	6	9.65103	6	2.99026	5	-.859136	5	-.234437	4	.81
.82	—	—	—	—	-.371123	7	-.133564										

TABLE 3-8. (continued)

x	F16(x)	p	F17(x)	p	F18(x)	p	F19(x)	p	F20(x)	p	F21(x)	p	F22(x)	p	F23(x)	p	i
0	.945531		0		.917148		0		.887746		0		.855653		0		0
.01	.851139		.452169		.791047		.457282		.753319		.461018		.720311		.46353		.01
.02	.813557		.790046		.459116		.78705		.40501		.781733		.556058		.774466		.02
.03	.819281	2	.928262		.430508	3	.897372		.622359	2	.864553		.11092		.83045		.03
.04	.375339		.831942		.450874		.757514		.511623		.584276		.55958		.613028		.04
.05	.743302		.52544		.787163		.406332		.511243		.795685		.821872		.133664		.05
.06	.933984		.816039	2	.915068		.518179	2	.276593		.183197		.873997		.28984		.06
.07	.902077		.375554		.800464		.5072		.68536		.607052		.765533		.678695		.07
.08	.655302		.743415		.473825		.816033		.275699		.847388		.178021		.845274		.08
.09	.72239		.925317		.214423	2	.899132		.181031		.831427		.350627		.734896		.09
.1	.211125		.875849		.437464		.733532		.606232		.563951		.719706		.38357		.1
.11	.625735		.607499		.782777		.365032		.857056		.125624		.86315		.939495	2	.11
.12	.893891		.187702		.922653		.104932		.861194		.35159		.755966		.541913		.12
.13	.952853		.280664		.827002		.547271		.61746		.724302		.37819		.814369		.13
.14	.789104		.680401		.506688		.840885		.195781		.880768		.94735	2	.822635		.14
.15	.441051		.913637		.587258	2	.905684		.283216		.774163		.547386		.565871		.15
.16	.104078	2	.923353		.405902		.724131		.682942		.436223		.826145		.121648		.16
.17	.461047		.707445		.763431		.345352		.889805		.326494	2	.849481		.361		.17
.18	.807555		.118671		.935371		.177824		.845255		.493574		.610063		.729791		.18
.19	.971157		.148751		.861238		.56866		.561904		.813485		.181552		.865193		.19
.2	.915185		.582343		.569955		.859055		.119395		.890164		.304541		.724102		.2
.21	.632758		.878363		.133166		.921803		.338483		.709248		.699509		.351297		.21
.22	.74305		.967347		.539153		.740467		.738721		.320475		.134946		.882938		.22
.23	.272658		.828191		.725027		.363024		.916066		.163314		.799062		.580805		.23
.24	.641731		.494192		.939883		.11151		.841769		.602366		.473111		.845688		.24
.25	.922909		.434252	2	.9201		.55957		.536319		.870518		.300312	3	.846301		.25
.26	.100591		.419927		.674961		.865397		.830047	2	.891155		.470647		.58224		.26
.27	.873734		.78999		.264417		.950714		.395464		.65846		.806878		.135408		.27
.28	.55531		.98331		.212374		.794139		.770547		.238206		.906141		.356099		.28
.29	.118234		.957186		.641543		.435654		.942323		.25165		.739179		.741076		.29
.3	.346347		.718256		.921833		.343386	2	.865568		.674608		.354764		.901665		.3
.31	.743212		.319824		.988142		.499446		.560761		.913736		.135478		.788878		.31
.32	.292605		.130953		.825851		.844731		.107581		.903543		.590165		.436633		.32
.33	.10458		.59302		.472966		.98665		.376826		.647022		.87914		.494719	2	.33
.34	.873736		.912994		.101038	2	.891553		.768639		.21365		.920367		.524622		.34
.35	.567355		.104496		.458829		.582929		.968967		.280472		.702328		.848257		.35
.36	.130122		.96305		.830418		.134192		.928153		.704401		.286051		.923555		.36
.37	.355353		.685358		.102441		.349769		.656965		.947124		.213141		.733444		.37
.38	.743554		.268538		.100034		.757776		.222412		.946051		.65839		.327894		.38
.39	.102185		.2048		.764694		.997926		.270048		.702005		.929004		.175335		.39
.4	.112791		.643496		.367857		.101763		.702913		.277426		.95263		.633773		.4
.41	.10317		.965584		.108114		.813904		.974826		.220976		.723421		.919026		.41
.42	.766228		.111324		.567484		.432151	2	.10237		.469999		.301952		.952204		.42
.43	.372478		.106214		.920551		.435003		.839651		.960642		.202376		.724497		.43
.44	.848991	2	.823939		.100088		.513095		.465255		.102339		.660627		.297873		.44
.45	.534781		.442031		.107681		.892338		.159677	2	.846238		.957761		.213405		.45
.46	.910286		.180038	2	.855407		.109206		.495574		.469878		.102083		.674512		.46
.47	.115953		.480954		.478814		.108137		.883559		.182948	2	.835473		.965926	2	.47
.48	.125153		.874998		.144249	2	.864951		.10897		.508699		.417282		.101373		.48
.49	.117875		.114284		.458657		.485564		.107839		.894318		.051436		.806806		.49
.5	.955717		.124927		.863944		.136185	2	.854493		.109385		.545175		.597339		.5
.51	.614307		.118429		.113997		.467509		.463587		.106766		.922743		.114489		.51
.52	.198352		.961931		.124882		.877145		.194047	2	.823667		.110201		.606621		.52
.53	.245438		.615584		.11797		.115089		.506348		.412807		.104639		.964938		.53
.54	.672037		.190821		.947728		.124984		.912944		.839145	2	.770123		.110924		.54
.55	.101427		.262185		.58869		.116434		.11733		.573221		.332105		.100959		.55
.56	.155589		.695994		.151331		.912319		.124854		.968019		.179107		.690007		.56
.57	.153281		.107125		.31119		.333356		.113228		.665056		.219273		.219926		.57
.58	.163257		.17617		.749166		.800401	2	.853215		.124284		.103717		.303461		.58
.59	.164243		.159249		.11224		.391387		.448045		.108765		.123248		.777102		.59
.6	.157663		.164242		.1403F6		.829468		.231777	2	.766572		.122258		.111297		.6
.61	.145341		.164022		.158074		.119582		.501277		.330678		.101502		.12536		.61
.62	.129723		.156182		.163345		.145665		.933627		.19819		.647816		.117983		.62
.63	.111147		.1427		.163295		.161352		.128145		.638443		.179335		.91005		.63
.64	.926843		.125643		.15373		.166175		.120209		.105783		.323931		.92454		.64
.65	.750492		.106921		.1388		.161722		.16456		.13795		.798974		.686678	3	.65
.66	.590765		.881296		.120685		.150049		.166249		.158443		.119535		.517559		.66
.67	.452445		.704654		.10134		.133483		.15891		.167089		.148032		.976877		.67
.68	.337319		.547138		.823474		.114291		.144865		.165027		.164127		.133825		.68
.69	.2449		.412868		.648461		.944388		.126605		.154452		.168179		.157552		.69
.7	.173173		.302923		.495351		.75448		.106446		.137934		.161932		.16829		.7
.71	.119266		.216158		.367303		.583534		.863072		.118083		.147983		.167174		.71
.72	.799905	2	.150024		.264475		.437303		.675969		.972051		.129087		.156519		.72
.73	.22307	2	.101266		.184953		.317706		.511979		.771076		.107929		.139234		.73
.74	.31896	2	.644599	2	.125612		.223822		.375257		.590239		.867235		.118283		.74
.75	.205131	2	.423914	2	.828313	2	.152904		.266263		.436388		.570897		.962868		.75
.76	.123232	2	.262647	2	.530117	2	.101271		.182907		.311775		.500249		.752731		.76
.77	.718989	3	.157955	2	.329086	2	.630003	2	.121621		.215278		.359753		.565919		.77
.78	.407010	3	.921256	3	.198005	2	.040406		.782456	2	.143641		.24958		.409506		.78
.79	.225294	3	.520538	3	.115361	2	.243064	2	.486748	2	.092574		.167012		.285295		.79
.8	.118565	3	.284583	3	.650062	3	.141347	2	.292518	2	.578877	2	.107751		.191343		.8
.81	.608387	4	.150318	3	.53809	3	.793591	3	.169633	2	.345443	2	.669731	2	.123484		.81
.82	.301134	4	.757594	4	.156691	3	.429520	3	.947912	3	.199563	2	.400607	2	.766168	2	.82
.83	.145483	4	.375528	4	.937968	4	.223695	3	.509543	3	.110836	2	.230283	2	.456488	2	.83
.84	.656499	5	.176839	4	.454963	4	.111839	3	.262939	3	.590976	3	.126986	2	.260756	2	.84
.85	.287614	5	.797422	5	.211344	4	.535587	4	.129932	3	.301646	3	.670268	3	.142513	2	.85
.86	.120217	5	.343204	5	.937218	5	.244928	4	.613033	4	.146998	3	.337727	3	.743346	3	.86

TABLE 3-8. (continued)

x	F ₂₄ (x)	p	F ₂₅ (x)	p	F ₂₆ (x)	p	F ₂₇ (x)	p	F ₂₈ (x)	p	F ₂₉ (x)	p	F ₃₀ (x)	p	F ₃₁ (x)	p	i
0	.830593		0		.80712		0		.784869		0		.763527		0		0
.01	.888729		.464926		.699614		.465278		.632279		.464626		.606478		.46298		.01
.02	.311539		.765329		.270969		.755129		.23386		.743411		.199891		.730472		.02
.03	.172187		.795556		.216877		.760249		.225665		.728221		.689028		.689494		.03
.04	.597439		.544346		.625775		.478641		.646112		.416207		.657471		.357249		.04
.05	.819117		.100553		.896417		.01633		.785793		.91578		.758935		.126147		.05
.06	.741586		.379223		.692792		.452618		.620356		.511359	2	.546469		.556781		.06
.07	.44435		.725736		.326204		.751653		.21383		.759718		.109144		.752943		.07
.08	2.47066	2	.816772		.159879		.768162		.276271		.704915		.373653		.631718		.08
.09	.484146		.620128		.588522		.495737		.660051		.568562		.703842		.243895		.09
.1	.783231		.204807		.803742		.3.64767	2	.788781		.115477		.745854		.247445		.1
.11	.815091		.283402		.727136		.437445		.612398		.554492		.482143		.63559		.11
.12	.570764		.673235		.386304		.748398		.198752		.773662		2.03026	2	.757128		.12
.13	.13268		.828014		.9.55214	2	.779794		.292724		.685542		.451011		.560582		.13
.14	.351432		.693384		.544063		.519566		.672602		.324705		.739202		.127977		.14
.15	.718524		.316076		.797266		6.44918	2	.794429		.166099		.726329		.359619		.15
.16	.845096		.172669		.763518		.416038		.610971		.592617		.417116		.697985		.16
.17	.488483		.603005		.45462		.743413		.192178		.785988		6.30856	2	.745207		.17
.18	.50081		.82555		.018631		.7.7701		.301841		.669395		.519363		.480928		.18
.19	.188809		.763026		.487367		.55315		.682228		.288213		.766424		.1.47673	2	.19
.2	.617595		.436632		.783443		.104894		.803537		.208054		.703663		.499452		.2
.21	.843273		4.15692	2	.801188		.340111		.619035		.624674		.355749		.744401		.21
.22	.791019		.507713		.533836		.733746		.19822		.798115		.137512		.721196		.22
.23	.477731		.802283		7.57216	2	.815674		.299884		.639938		.577671		.398609		.23
.24	5.92779	3	.824586		.411513		.599266		.686926		.263301		.78758		.9.06964	2	.24
.25	.47034		.566629		.756208		.162866		.816753		.23027		.682274		.545092		.25
.26	.796028		.115289		.837034		.335255		.639853		.649631		.303054		.777157		.26
.27	.86539		.377591		.625198		.714634		.221789		.811845		.199481		.690713		.27
.28	.657736		.746623		.19396		.837927		.281788		.661298		.625537		.320145		.28
.29	.234034		.868279		.307416		.660142		.68349		.25501		.805617		.183879		.29
.3	.265294		.701683		.705763		.244288		.833899		.251894		.667557		.616098		.3
.31	.483744		.301489		.863788		.26174		.676664		.658665		.264756		.800291		.31
.32	.889078		.200945		.726857		.678262		.268804		.828833		.245204		.660723		.32
.33	.816604		.641291		.340991		.857634		.240799		.780704		.662688		.252561		.33
.34	.488621		.876117		.163567		.736048		.66631		.269508		.824168		.260688		.34
.35	6.45567	3	.829117		.617014		.3553		.852743		.24392		.665756		.673164		.35
.36	.481762		.514943		.867295		.152601		.731872		.670084		.247597		.819212		.36
.37	.827015		5.34605	2	.830619		.611962		.346209		.849604		.26988		.659348		.37
.38	.924646		.462953		.518496		.864369		.167018		.714869		.688108		.205543		.38
.39	.745169		.818294		.3.28879	2	.822817		.625487		.314453		.846569		.316928		.39
.4	.341848		.92139		.468187		.500626		.86673		.205515		.683754		.71737		.4
.41	.166466		.740066		.822503		.5.88853	3	.805146		.655471		.259918		.840099		.41
.42	.631449		.329592		.9186045		.49599		.461222		.871659		.266366		.635788		.42
.43	.918705		.18598		.718515		.83794		.4.66495	2	.773072		.698304		.182094		.43
.44	.946215		.650604		.29172		.906668		.54406		.399075		.874502		.347024		.44
.45	.706332		.927061		.231323		.678976		.860926		.123562		.72849		.748696		.45
.46	.266175		.533509		.588907		.277985		.885194		.608638		.312424		.868826		.46
.47	.252869		.688025		.940847		.301931		.618412		.885803		.223015		.660257		.47
.48	.709787		.204888		.910634		.742403		.137829		.857746		.684239		.199781		.48
.49	.982505		.319124		.607059		.95464		.394632		.532909		.904912		.341616		.49
.5	.999687		.762974		.119631		.872183		.805212		.2.11606	2	.805092		.763161		.5
.51	.757663		.1.0026		.409643		.519898		.960898		.504954		.418533		.908688		.51
.52	.318911		.973451		.828699		.4.33251	3	.811279		.669164		.120628		.723363		.52
.53	.2063		.683729		1.01903		.320331		.402828		.95013		.625917		.272484		.53
.54	.889245		.210504		.927889		.899286		.13778		.720323		.923531		.283471		.54
.55	1.01484		.324777		.580673		1.0224		.644688		.253106		.911357		.747273		.55
.56	1.10832		.787288		7.11958	2	.894565		.964376		.302972		.592112	2	.955068		.56
.57	.950447		1.06397		.465967		.443148		.1.00137		.772961		.7.04217		.633057		.57
.58	.578655		1.08986		.893865		.9.81593	2	.74467		.1.01078		.483704		.421445		.58
.59	7.51537	2	.860957		.1.10127		.622987		.268166		.743333		.891502		.141381		.59
.6	.455862		.431572		.1.04278		.992871		.293498		.599429	2	1.02282		.66743		.6
.61	.902239		.1.01766		.73289		.1.13518		.78497		.5.58397		.835777		.982634		.61
.62	.1.1824		.824566		.2.45732		.959024		1.07366		.505018		.387081		.983469		.62
.63	.1.25499		.1.03028		.307466		.559184		1.08424		.566275		.188406		.668412		.63
.64	.1.10439		.1.24171		.806127		.2.12294	2	.613563		1.11654		.720813		.135419		.64
.65	.765241		.1.22373		1.14743		.534185		.335863		.598473		.1.0563		.451795		.65
.66	.297127		.965683		1.2665		.984475		.236823		.614823		.1.10042		.916081		.66
.67	.226726		.574411		1.14594		1.23621		.768393		6.44002	2	.841707		.1.12034		.67
.68	.733674		.6.08654	2	.814047		1.24254		.1.14045		.516356		.350266		.1.00381		.68
.69	1.16358		.475887		.333816		1.00821		.1.27599		.989834		.245777		.604555		.69
.7	1.47629		.963575		.213511		.582557		.1.15226		.1.25014		.796775		.2.74171	2	.7
.71	1.65416		1.34771		.745838		4.42109	2	.799708		.1.24464		1.1715		.574421		.71
.72	1.70033		1.59753		.1.19483		.518041		.289215		.98021		1.28646		1.04837		.72
.73	1.63387		1.70544		.1.51524		.1.02321		.287467		.51493		1.12129		1.28105		.73
.74	1.48344		1.68504		.1.68813		.1.41137		.838605		.06067		.716422		1.22226		.74
.75	1.28099		1.56293		.1.71867		.1.65025		1.28974		.6166		.15677		.89042		.75
.76	1.05451		1.3724		.1.63012		.1.75357		1.59427		1.15461		.452391		.360392		.76
.77	.834546		1.1469		.1.4561		.1.68488		1.73641		1.52307		.1.01022		.260566		.77
.78	.632449		.915222		.1.23297		.1.33151		1.72759		1.72351		.1.43981		.860661		.78
.79	.460282		.698879		.993753		.1.31335		1.59844		1.7991		.1.69881		.1.34775		.79
.8	.321835		.511299		.764309		.1.06928		1.38931		1.65715		.1.78065		.1.66467		.8
.81	.216182		.358563		.561759		.827882		1.1411		1.45883		.1.70813		.1.79381		.81
.82	.13943		.241023		.394813		.610872		.888824		1.20868		.1.52249		.1.75229		.82
.83	.862684	2	.153202		.265322		.429885		.637868		.94644		.1.27134		.1.58054		.83
.84	5.11306	2	.035637		.170377		.2885		.465074		.702		.1.00012		.1.32976		.84
.85	2.89788	2	.563068	2	.104417		.184502		.309661		.49367		.742546		.1.0493		.85
.86	1.56683	2	.5.16072		.6.07627	2	.1.12277		.197109		.32909						

TABLE 3-8. (continued)

ν	$F_{32}(x)$	p	$F_{33}(x)$	p	$F_{34}(x)$	p	$F_{35}(x)$	p	$F_{36}(x)$	p	$F_{37}(x)$	p	$F_{38}(x)$	p	$F_{39}(x)$	p	i
0	.74281		0		.722438		.702111		0		.68147		0		0		0
.01	.581886		.46032		.558274		.456584		.555293		.451656		.512965		.445335		.01
.02	.16879		.716352		.140338		.701032		.114361		.684413		.090724		.646279		.02
.03	-.317615		.654427		.612718		.541555		.36116		.783405		.376561		.551439		.03
.04	-.665704		.301912		.66251		.250299		.66544		.202486		.657882		.194542		.04
.05	-.727256		.184918		.691934		.235727		.653359		.278924		.614052		.314597		.05
.06	-.472681		.590176		.403986		.612732		.331893		.625599		.266492		.625635		.06
.07	-.134368	2	.73406		.724781	2	.705509		.14813		.669439		.213275		.627695		.07
.08	.452489		.552536		.51371		.470679		.558526		.388886		.588279		.309402		.08
.09	.72342		.125711		.722451		.1.68887	2	.704537		8.05688	2	.673083		.165324		.09
.1	.682033		.557626		.603727		.445585		.516565		.511562		.425379		.557631		.1
.11	.345806		.685547		.210907		.702232		8.32187	2	.696155		.3.30204	2	.669856		.11
.12	-.140541		.707597		.278427		.63579		.539056		.543838		.476149		.445		.12
.13	-.567456		.41865		.642831		.271549		.680459		.127963		.685253		.4.44907	3	.13
.14	-.750841		.5.61672	2	.717151		.217795		.648359		.350934		.556668		.452868		.14
.15	-.611287		.507169		.467195		.607525		.310026		.661559		.153041		.674745		.15
.16	-.208191		.736377		.5.22267	3	.717355		.176534		.653464		.327326		.597972		.16
.17	.28578		.641263		.460668		.495935		.581272		.329755		.647949		.160315		.17
.18	.658806		.703341		.720324		.4.45837	2	.712859		.154271		.65049		.319317		.18
.19	.750687		.251898		.656194		.429069		.508444		.565727		.332748		.640396		.19
.2	.521381	2	.627234		.796254		.70478		6.33931	2	.706179		.150009		.641315		.2
.21	6.80664		.748963		.198509		.66009		.41343	2	.507128		.561052		.320515		.21
.22	-.416376		.542954		.605309		.309435		.697761		6.31287	2	.678245		.1.62411		.22
.23	-.725578		9.85149	2	.743955		.185115		.653272		.413275		.493241		.566241		.23
.24	-.727453		.391517		.54888		.597147		.30472		.693572		.4.52335	2	.688572		.24
.25	-.420217		.711074		.107153		.737546		.19066		.642529		.427276		.467053		.25
.26	6.69377	2	.725859		.384877		.614964		.541013		.283224		.693112		.1.08321	2	.26
.27	.528568		.423066		.708038		.9.59981	2	.72987		.213737		.621357		.453363		.27
.28	.769725		6.51156	2	.717821		.592506		.519935		.611447		.245492		.694094		.28
.29	.688016		.528159		.408568		.709816		.065979		.719288		.25256		.590271		.29
.3	.316906		.765978		.8.39823	2	.703198		.421682		.485189		.630475		.191782		.3
.31	-.190277		.675128		.542692		.377142		.071601		.081093		.704162		.304609		.31
.32	-.621069		.235203		.764844		.122193		.68055		.460892		.43563	2	.653587		.32
.33	-.75957		.217358		.651412		.269906		.328489		.724496		.4.64861		.680144		.33
.34	-.641142		.658797		.250574		.763705		.178122		.646584		.509706		.369872		.34
.35	-.219218		.790923		.607675		.666319		.606435		.262002		.729674		.125995		.35
.36	.296809		.164729		.781838		.32603		.75862		.249524		.598541		.563596		.36
.37	.694992		.351401		.558077		.690339		.563026		.647713		.177291		.726577		.37
.38	.811118		.724406		9.00725	2	.764076		.40188		.744553		.333119		.53271		.38
.39	.597058										.492509		.687893	2	.7.48086		.39
.4	.13741		.796631		.421351		.485501		.732165		.007258		.715649		.424174		.4
.41	.382539		.556115		.756127		.4.91187	3	.732011		.486138		.400662		.71988		.41
.42	.753276		.4.93918	2	.769396		.502195		.359149		.757732		.109152		.669599		.42
.43	.825002		.462996		.453373		.783328		.18117		.679545		.572209		.285888		.43
.44	-.567301		.78954		.5.94866	2	.723396		.587638		.285094		.747804		.337766		.44
.45	-.8.07358	2	.734753		.552879		.346102		.79774		.248795		.400857		.651535		.45
.46	.443537		.474386		.818139		.186551		.632123		.665346		.581377		.491437		.46
.47	.799445		.4.32847	2	.742008		.644592		.121974		.790035		.381377		.491437		.47
.48	.846695		.5499		.553835		.829471		.376595		.549851		.736637		.1.15416	2	.48
.49	.564957		.841283		.186991		.659417		.728083		.5.52167	2	.750615		.515395		.49
.5	6.09162	2	.799192		.657454		.204316		.812885		.471155		.412939		.717965		.5
.51	-.47355		.45796		.865038		.344276		.54083		.790944		.122199		.470192		.51
.52	-.835136		.101917		.717354		.75454	2	.777481		.757729		.608074		.241374		.52
.53	-.886067		.609742		.276812		.852211		.505024		.362889		.819128		.309552		.53
.54	-.605166		.818276		.282692		.593489		.826635		.169296		.654999		.721697		.54
.55	9.45498	2	.825377		.737272		.8.29112	2	.796252		.654657		.186915		.794445		.55
.56	.459391		.445164		.904602		.470445		.42339		.45726		.374436		.499584		.56
.57	.855041		.715897		.839365		.136717		.136717		.684591		.774464		.039203		.57
.58	.948627		.635445		.24212		.871376		.648569		.208117		.829885		.569022		.58
.59	.704763		.951631		.334797		.55015		.896299		.367916		.51143		.843084		.59
.6	.207203		.888599		.793213		.1.24634	3	.773469		.795049		.4.28031	2	.730947		.6
.61	-.372508		.519416		.954932		.558919		.326842		.887573		.586689		.274993		.61
.62	-.835857		.4.52359		.761586		.909292		.263578		.601765		.884122		.310266		.62
.63	-.1.0254		.605721	2	.276385		.910707		.758693		.5.40826	2	.795555		.770015		.63
.64	-.8765		.96512		.32162		.558553		.957336		.52835		.356349		.849821		.64
.65	-.43615		.997611		.815716		.1.51776	2	.76505		.90295		.247836		.610487		.65
.66	.155476		.689889		.1.02745		.597152		.763189		.91205		.759186		.5.07543	2	.66
.67	.712624		.1.42733		.879459		.970109		.333384		.346947		.957942		.545202		.67
.68	.1.06348		.465624		.420284		.394835		.836453		.4.96592	2	.75524		.315255		.68
.69	.1.10202		.93986		.19554		.658843		.1.03283		.642156	2	.229773		.893818		.69
.7	.816627		.1.13029		.762708		.7.80805	2	.845011		.996249		.405791		.483403		.7
.71	.288698		.97757		.1.09479		.54698		.33786		.970485		.89273		.148333		.71
.72	-.355692		.52652		.1.084		.1.00286		.30805		.369405		.1.05177		.735425		.72
.73	-.890076		.092524		.732097		.1.13561		.863995		.6.06383	2	.761709		.1.02956		.73
.74	-.1.2334		.706258		.14466		.899517		.1.13492		.688883		.181967		.907202		.74
.75	-.1.28482		.1.14963		.506245		.36728		.1.02554		.1.08663		.488512		.4034		.75
.76	-.1.03323		.1.0364		.1.0364		.298612		.568723		.1.10925		.996874		.273517		.76
.77	-.54691		.1.15055		.1.30237		.900526		.9.08932	2	.744309		.1.15164		.872616		.77
.78	.7.49851	2	.714189		.1.2373		.1.26646		.748555		.110795		.891137		.1.1551		.78
.79	.709665		.1.101285		.86116		.1.29731		.1.20662		.586424		.301422		.1.00783		.79
.8	.1.25014		.560544		.265982		.987697		.1.3331		.1.12753		.419436		.47741		.8
.81	.1.62366		.1.19006		.416115		.417611		.1.09446		.1.34762		.1.03367		.251949		.81
.82	.1.80056		.1.57232		.1.05042		.270661		.35938		.1.16274		.1.34416		.929145		.82
.83	.1.79049		.1.8023		.1.53146		.953841		.120044		.679105		.1.25427		.1.32585		.83
.84	.1.63338		.1.82421		.1.80169		.1.48544		.862637		.3.13429	2	.789114		.31111		.84
.85	.1.38289		.1.68156		.1.85475		.1.80068		.1.44282		.779084		.7.88945	2	.886139		.85
.86	.1.09349		.1.43098		.1.72572		.1.8835										

TABLE 3-8. (continued)

x	F ₄₀ (x)	p	F ₄₁ (x)	p	F ₄₂ (x)	p	F ₄₃ (x)	p	F ₄₄ (x)	p	F ₄₅ (x)	p	F ₄₆ (x)	p	F ₄₇ (x)	p	x
0	.660332		0		.637039		0		.611028		0		.578217		0		0
.01	.450633		.437269		.467865		.42678		.443648		.412266		.415368		.38939		.01
.02	.633333	2	.646203		.501397	2	.623317		.531326	2	.595475		.184968	2	.356752		.02
.03	.387731		.517632		.394389		.483509		.395675		.447755		.388955		.406581		.03
.04	.646015		.11875		.629674		.8.26368	2	.607934		.5.10387	2	.577505		.024366		.04
.05	.572847		.342793		.530654		.363166		.487202		.374372		.4408		.372059		.05
.06	.205514		.625513		.14959		.615534		.7.93391	2	.592074		.5.55631	2	.356589		.06
.07	.267169		.581792		.311387		.532		.343438		.480854		.56145		.423756		.07
.08	.604279		.254082		.607602		.164499		.598617		.102119		.575354		.4.88829	2	.08
.09	.63119		.256469		.581538		.293195		.52615		.334009		.465341		.354474		.09
.1	.334236		.584378		.246535		.593474		.165163		.585218		9.28581	2	.556255		.1
.11	.134821		.62782		.270091		.574053		.287055		.511545		.352718		.440955		.11
.12	.535777		.345345		.570889		.244785		.582979		.15328		.571177		7.37048	2	.12
.13	.662939		.12078		.613473		.217387		.560169		.291087		.48906		.336516		.13
.14	.450529		.523309		.539241		.563499		.230359		.574799		.130929		.555731		.14
.15	.656681	3	.653332		.121817		.606607		.226613		.539764		.302602		.458337		.15
.16	.442144		.443816		.519661		.322539		.560853		.204552		.566516		.5.89936	2	.16
.17	.665866		.1.53751	3	.643116		.136276		.588963		.245477		.512005		.318048		.17
.18	.549436		.443114		.425881		.525334		.294627		.560647		.168736		.534989		.18
.19	.151347		.698499		.1.80097	2	.628717		.162177		.565396		.270913		.475978		.19
.2	.325765		.531775		.454126		.397122		.532003		.255955		.559634		.124922		.2
.21	.63832		.127423		.651418		.5.04406	2	.612152		.197015		.554202		.298921		.21
.22	.626041		.344353		.504489		.47282		.357613		.542529		.277645		.553987		.22
.23	.293929		.64007		.8.97099	2	.642542		9.36008	2	.582827		.237429		.495978		.23
.24	.189871		.603771		.374599		.467172		.496114		.307445		.550913		.15164		.24
.25	.579513		.253457		.643095		.5.94697	2	.629132		.45499		.555654		.273959		.25
.26	.675524		.230425		.572917		.411776		.1.18889		.520134		.247429		.552668		.26
.27	.428154		.597414		.19948		.644119		.2.15412	2	.608132		.201886		.51244		.27
.28	.5.66727	2	.456608		.28156		.531563		.452793		.359125		.540149		.179478		.28
.29	.488696		.375715		.616882		.132689		.639333		.9.06899	2	.576531		.25796		.29
.3	.49325		.101661		.628788		.340017		.477921		.493067		.288275		.550959		.3
.31	.547158		.529601		.30912		.633345		.054328		.16397		.15347		.53147		.31
.32	.122467		.616537		.175714		.588877		.410559		.410805		.527057		.208556		.32
.33	.367377		.489725		.5715		.228265		.641893		3.23053	2	.596168		.235683		.33
.34	.674923		.5.84923	2	.669437		.257373		.534037		.460827		.330581		.548761		.34
.35	.643549		.436123		.416024		.608952		.134291		.637355		.123308		.551184		.35
.36	.286829		.689775		.058093		.637387		.341837		.462409		.511303		.239153		.36
.37	.217622		.590336		.50569		.325077		.635835		2.99792	2	.615053		.211836		.37
.38	.616824		.116334		.691862		.163738		.586378		.42279		.373965		.545943		.38
.39	.709231		.317076		.51704		.569458		.217559		.645759		7.94469	2	.571704		.39
.4	.444084		.664259		.6.98079	2	.675033		.272953		.513751		.492455		.271717		.4
.41	.434748	2	.671567		.418279		.421553		.619927		9.65335	2	.632809		.186063		.41
.42	.516182		.536962		.692661		5.91207	2	.633829		.376173		.419256		.542331		.42
.43	.735599		.173433		.608293		.513297		.304022		.648973		.3.18372	2	.593164		.43
.44	.590021		.600773		.205833		.699292		.194129		.56524		.469876		.306321		.44
.45	.148679		.726586		.308298		.51572		.592636		.167772		.649175		.15792		.45
.46	.371742		.485998		.667987		5.62504	2	.674926		.326		.466505		.557694		.46
.47	.713762		.7.44807	3	.68498		.435055		.393131		.646013		.2.01967	2	.615711		.47
.48	.706254		.101175		.346883		.70705		.104487		.614266		.442876		.344087		.48
.49	.349371		.747398		.175175		.604913		.592045		.244149		.663767		.126842		.49
.5	.184623		.619097		.613519		.181781		.707728		.265037		.515976		.531717		.5
.51	.655035		.174748		.741087		.543031		.482901		.635512		7.74946	2	.659502		.51
.52	.77959		.508117		.485308		.694437		.5.06663	3	.462274		.1.41047		.365011		.52
.53	.545068		.725		.1.90605	2	.685112		.495753		.326142		.676027		.9.20989	2	.53
.54	.403827	2	.73138		.524491		.316263		.729621		.193769		.56794		.523965		.54
.55	.491874		.372256		.769039		.224637		.575815		.621532		.141114		.664709		.55
.56	.795974		.17828		.624292		.656606		.111128		.707481		.371271		.430053		.56
.57	.71695		.649389		.158617		.752533		.420726		.414223		.685137		5.27231	2	.57
.58	.292857		.8067		.395846		.457474		.737034		.110201		.622655		.513827		.58
.59	.275382		.568094		.75927		8.10713	2	.669594		.583864		.212407		.691508		.59
.6	.728869		.4.51496	2	.744969		.587465		.239018		.748261		.323549		.480155		.6
.61	.839451		.510052		.354223		.799011		.323123		.50878		.689681		.7.40362	3	.61
.62	.553831		.826828		.222651		.501512		.725011		.1.18256	2	.680331		.50042		.62
.63	.533644	4	.747226		.699728		8.96142	2	.751317		.536712		.293127		.720069		.63
.64	.564813		.303248		.837705		.475361		.391221		.782088		.264782		.536738		.64
.65	.878828		.297204		.560308		.215064		.198066		.609971		.688411		.4.56897	2	.65
.66	.791505		.768671		.1.05508	3	.741639		.686538		.10461		.741039		.482434		.66
.67	.354213		.883674		.575432		.288348		.826536		.469206		.385995		.750922		.67
.68	.284979		.580948		.880941		.527757		.537558		.804958		.191385		.601461		.68
.69	.792329		.4.69807	3	.762786		.786491		.5.93922	2	.717404		.67784		.109199		.69
.7	.955219		.604115		.269539		.866248		.611473		.243332		.804532		.457844		.7
.71	.693614		.938557		.568417		.515492		.582718		.374075		.492938		.742877		.71
.72	.115428		.482774		.847715		.108247		.705984		.810325		9.80453	2	.676861		.72
.73	.531172		.363395		.935846		.492338		.160522		.829444		.653479		.187101		.73
.74	.579888		.30209		.581672		.955114		.484794		.409815		.869837		.42338		.74
.75	.1.00152		.853735		.5.91782	2	.757403		.904635		.240323		.619451		.816448		.75
.76	.669438		.1.04309		.695223		.182428		.880198		.787024		2.34729	2	.766757		.76
.77	.5.56936	2	.777986		.1.03256		.501894		.410492		.941626		.607308		.285704		.77
.78	.721052		.622296		.908866		.972184		.283663		.61096		.934305		.373432		.78
.79	.1.12298		.549185		.367029		.998415		.865156		.5.96352	2	.771129		.251387		.79
.8	.1.09424		.1.05908		.363473		.554227		.1.04392		.714955		.186628		.876276		.8
.81	.525316	2	.1.5107		.947291		.166642		.718467		.1.04274		.524652		.415715		.81
.82	.8.74164		.776709		.1.17953		.85105		.2.74309	2	.854547		.991144		.297582		.82
.83	.817491		.7.16316	2	.89794		.1.18086		.713124		.223624		.995609		.883436		.83
.84	.1.29578		.70139		.223561		.599856		.1.15585		.574914		.415123		.1.01503		.84
.85	.1.35347		.1.23671		.563541		.367574		.1.08242		.1.10398		.375976		.597095		.85
.86	.971219		.1.38957		.1.21094		.464525		.506551		.1.14507		.1.02221				

TABLE 3-8. (continued)

x	F ₄₈ (x)	p	F ₄₉ (x)	p	F ₅₀ (x)	p	F ₅₁ (x)	p	F ₅₂ (x)	p	F ₅₃ (x)	p	F ₅₄ (x)	p	F ₅₅ (x)	p	i
0	.532277		0		.510897		.510897		.543807		0		.570458		0		0
.01	.5378751		.561887		.560526		.574244		.578754		.403051		.395182		.417723		.01
.02	.5638361	3	.512892		.512206	3	.525986		.5172417		.560958		.527554		.574905		.02
.03	.5638389		.564946		.563666		.56483		.406233		.377575		.426053		.373428		.03
.04	.532539		.413130	3	.511287		.513298	2	.554315		.57528	2	.566311		.612008	2	.04
.05	.53885		.559382		.557932		.583523		.572452		.427764		.56939		.45795		.05
.06	.501004	2	.513683		.638325	3	.526487		.504699	2	.559949		.734032	2	.569181		.06
.07	.530346		.568983		.567378		.555928		.412331		.5516		.499809		.525127		.07
.08	.533326		.529524	3	.512464		.026696		.543434		7.09072	2	.564089		.122348		.08
.09	.598711		.597412		.555367		.594046		.344527		.450691		.521525		.494172		.09
.1	.3.36671	2	.515278		.510696		.527492		.6.53108	2	.556427		.511894		.558001		.1
.11	.531554		.572832		.571697		.547157		.459903		.323159		.486164		.273347		.11
.12	.534648		1.25253	2	.514444		.4.02921	2	.553754		.107819		.553445		.182711		.12
.13	.409462		.555969		.554318		.404689		.537723		.474484		.525569		.540588		.13
.14	.4.74882	2	.517698		1.50966	2	.529011		.078992		.553297		.169748		.217419		.14
.15	.342947		.577756		.538555		.541935	2	.446047		.295442		.515377		.242671		.15
.16	.536517		.516857		.576667		.415939		.542241		.142915		.544591		.552763		.16
.17	.490841		.55505		.553281		.551059		.508743		.495089		.238323		.525127		.17
.18	.6.16754	2	.52098		.519623	2	.531172		.514938		.545988		.215451		.517776		.18
.19	.534458		.583348		.583348		.583348		.474228		.26341		.536933		.158504		.19
.2	.538952		2.13288	2	.52094		.6.85131	2	.552351		.181597		.5266912		.501352		.2
.21	.432948		.534659		.532759		.527891		.501057		.518175		.5266912		.501352		.21
.22	.5076551		.525174		.525174	2	.533656		.501057		.518175		.5266912		.501352		.22
.23	.326014		.589737		.588793		.521582		.52957		.540411		.264512		.487967		.23
.24	.541979		.525555	2	.525555		8.33843	2	.52957		.540411		.264512		.487967		.24
.25	.443896		.54808		.527462		.440657		.539983		.217172		.510732		.539342		.25
.26	9.16521	2	.530345		.502923		.536827		.52928		.536487		.144551		.590782		.26
.27	.517936		.596986		.5396101		.513059		.509967		.528756		.512285		.452582		.27
.28	.54963		.508696	2	.551172		.589451	2	.54955		.156779		.577268		2.94172	2	.28
.29	.459816		.53552		.553306		.454361		.54955		.238319		.485001		.411505		.29
.3	.5107736		.536579		.034419		.540605		.183654		.520097		.56357		.409143		.3
.31	.508938		.405198		.40437		.504395		.183654		.520097		.56357		.409143		.31
.32	.549948		.036044		.537888		.51364		.5619		.163367		.595716		4.10054	2	.32
.33	.474864		.55826		.55442		.469155		.227655		.294952		.468613		.460245		.33
.34	.124786		.543987		.5.99429	2	.545029		.224561		.574723		3.86684	2	.604066		.34
.35	.500117		.4145		.415742		.295488		.545926		.505052		.409201		.559705		.35
.36	.554982		.545822	2	.545822		.545822		.545663		.53396		.609009		.113331		.36
.37	.491226		.558767		.558141		.558141		.545663		.53396		.609009		.113331		.37
.38	.413022		.552706		.4.58637	2	.550148		.216446		.596357		.424826		.518281		.38
.39	.290953		.425052		.42434		.286213		.243072		.490311	2	.2.23857	2	.598493		.39
.4	.560795		.4.75416	2	.555128		.151605		.530026		.377646		.389573		.565817		.4
.41	.509127		.561401		.558517		.50276		.17877		.609036		.589573		.565817		.41
.42	.162711		.562915		.523378	2	.556021		.288375		.46625		.50208	2	.565817		.42
.43	.281297		.437055		.436403		.276418		.589007		.46625		.50208		.235225		.43
.44	.567463		.5.40475	2	.555999		.171954		.538758		.3.32627	2	.561436		.2685		.44
.45	.528846		.564904		.561616		.52205		.538758		.426233		.541862		.6047		.45
.46	.184184		.574841		.5.94268	2	.562716		.164316		.629345		.150767		.561801		.46
.47	.270965		.450765		.450175		.562716		.510302		.447687		.555404		.158369		.47
.48	.575078		.6.12191	2	.576687		.19425		.597475		.011082		.617198		.551127		.48
.49	.550751		.569075		.565523		.543417		.520034		.466844		.293051		.642972		.49
.5	.207842		.588777		.6.73056	2	.570318		.56128		.413897		.597889		7.39348	2	.5
.51	.259714		.466513		.465989		.254451		.56128		.413897		.597889		7.39348	2	.51
.52	.583753		.6.92211	2	.593517		.218957		.527294		.7.12686	2	.6047		.432992		.52
.53	.575226		.575226		.575226		.575226		.520848		.520848		.229116		.668646		.53
.54	.234289		.575226		.575226		.575226		.520848		.520848		.229116		.668646		.54
.55	.247228		.635109		7.61777	2	.578926		.588948		.653391		.297586		.477622		.55
.56	.593627		.484735		.484282		.241704		.64372		.124732		.645004		2.39711	2	.56
.57	.602908		.7.82701	2	.610917		.246685		.503552		.564216		.590527		.518239		.57
.58	.264187		.624352		.7.62435	2	.594173		4.76433	2	.656742		.162014		.686356		.58
.59	.233069		.506014		.565523		.588657		.450016		.338431		.37517		.414084		.59
.6	.604871		.08866		.631464		.27824		.507809		.624827		.7.56707	2	.686442		.6
.61	.634547		.588623		.588339		.624815		.507809		.624827		.7.56707	2	.686442		.61
.62	.298538		.647209		9.80971	2	.599645		.484632		.649204		.465792		.32774		.62
.63	.216626		.531156		.530872		.210382		.695173		.297151		.721017		.252424		.63
.64	.617693		.1.008		.655956		.31475		.475395		.26375		.52049		.679814		.64
.65	.671195		.398194		.392085		.660173		.475395		.26375		.52049		.679814		.65
.66	.338727		.674669		.674669		.660173		.475395		.26375		.52049		.679814		.66
.67	.196999		.561308		.561308		.612041		.560065		.226165		.54158		.221934		.67
.68	.632357		.115283		.115283		.190255		.741884		.362558		.741884		.382869		.68
.69	.714338		.409953		.402667		.57809		.473207	2	.739142		.15847		.748071		.69
.7	.586773		.70817		.129003		.626002		.606869		.16078		.764358		.529559		.7
.71	.172821		.598158		.598099		.626002		.606869		.16078		.764358		.529559		.71
.72	.649172		.132998		.721834		.626002		.606869		.16078		.764358		.529559		.72
.73	.766161		.424584		.413673		.751046		.424324		.786329		.266521		.54958		.73
.74	.445765		.749899		.150172		.641631		.186047		.623682		.752743		.8.40519	2	.74
.75	.141891		.644326		.644412		.135693		.705719		.4.99278	2	.755486		.679525		.75
.76	.668502		.155363		.767476		.474364		.805175		.579607		.255767		.830935		.76
.77	.830046		.443147		.431882		.811487		.406937		.86007		.433236		.412052		.77
.78	.520712		.803376		.177635		.658973		.775453		.596273		.859316		.278368		.78
.79	.10043		.704125		.704391		.5.11583		.810447		.6.17079	2	.726053		.53748		.79
.8	.690708		.184782		.826716		.557681		.521666		.897944		.619827		.222894		.8
.81	.911582		.467413		.452504		.887691		.404935		.503365		.619827		.222894		.81
.82	.420395	2	.674726		.215115		.677502		.514423		.252261		.631504		.551959		.82
.83	.4.13438		.785293		.785774		3.06485	2	.866608		.88407		.136216		.768683		.83
.84	.715924		.225742		.907228		.670835		.261376		.560258		.59112		.708289		.84
.85	.1.0209		.500634		.479528		.987937		.539494		.394894		.8.22236	2	.8.22236		.85
.86	.761994	2	.975829		.7014		.695287		1.0219		.468325		1.0116		.456		

REFERENCES

- Flammer, C. (1957). Spheroidal Wave Functions. Stanford University Press, Stanford, California.
- Fong, T. S. (1965). On the Problem of Antenna Pattern Synthesis and Optimum Aperture Distribution under Constraint. Scientific Report No. 6 on Contract AF19(628)-4349. Hughes Aircraft Company, Report No. P66-12, AFCRL-66-138, Culver City, California, December 1965.
- Rhodes, D. R. (1963). The optimum line source for the best mean-square approximation to a given radiation pattern. IEEE Trans. AP-11, 440-446.
- Stratton, J. A., P. M. Morse, L. J. Chu, J. D. C. Little, and F. J. Corbato (1956). Spheroidal Wave Functions. The Technology Press, Massachusetts Institute of Technology, Cambridge, Massachusetts.

RELATED CONTRACTS AND PUBLICATIONS

The following contracts relate to the study of antenna radiation problems in aircraft and space vehicles.

AF 19(604)-8386

AF 19(604)-3508

The following related reports have been published on the present contract:

J. H. Harris and J. Pachares, The Computation of Cylindrical Plasma Radiation Problems. Scientific Report No. 1 under Contract AF 19(628)-4349. Hughes Aircraft Company, Report No. P65-79, AFCRL-65-610, June 1965.

J. H. Harris, Radiation Pattern Characteristics of Line Sources in the Presence of Dielectric Layers. Scientific Report No. 2 under Contract AF 19(628)-4349. Hughes Aircraft Company, Report No. P65-91, AFCRL-65-666, July 1965.

J. H. Harris, The Nature of Leaky Waves in Multiply Layered Media. Scientific Report No. 3 under Contract AF 19(628)-4349. Hughes Aircraft Company, Report No. P65-96, AFCRL-65-667, August 1965.

J. H. Harris, Leaky Waves and Their Application to Single Aperture Multiple Beam Antennas. Scientific Report No. 4 under Contract AF 19(628)-4349. Hughes Aircraft Company, Report No. P65-99, AFCRL-65-672, August 1965.

A. T. Villeneuve, Optimization of Gain of Arbitrary Array Antennas. Scientific Report No. 5 under Contract AF 19(628)-4349. Hughes Aircraft Company, Report No. P65-153, AFCRL-66-67, December 1965.

Thomas S. Fong, On the Problem of Antenna Pattern Synthesis and Optimum Aperture Distribution under Constraint. Scientific Report No. 6 under Contract AF 19(628)-4349. Hughes Aircraft Company, Report No. P66-12, AFCRL-66-138, December 1965. To be published in the Journal of the Franklin Institute.

B. J. Forman, A. F. Seaton, C. H. Nonnemaker, Jr., An Investigation of a Variable Waveguide Coupler with 360 Degrees of Phase Control. Scientific Report No. 7 under Contract AF 19(628)-4349. Hughes Aircraft Company, Report No. P67-01, AFCRL-67-0090, November 1966.

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APPENDIX

A. PROGRAM FOR COMPUTATION OF THE EIGENFUNCTIONS AND THE EIGENVALUES

```

100 READ K,G
110 LET A(151)=-G
120 LET P=3.14159265
130 LET C=10*P
140 PRINT "C="C,"K="INT(K);K-INT(K)
150 LET W=C/2
160 DIM A (500),Z(300),P(150)
170 LET A(149)=1E-40
180 DEF FNC(N)=-N*(N-1)*W/(2*N-3)/(2*N-1)
190 DEF FNB(N)=N*(N+1)-K+(2*N*(N+1)-1)*W/(2*N-1)/(2*N+3)
200 DEF FNA(N)=(N+2)*(N+1)*W/(2*N+3)/(2*N+5)
210 FOR N=149 TO 2 STEP -2
220 LET A(N-2)=(FNB(N)*A(N)+FNA(N)*A(N+2))/FNC(N)
230 NEXT N
240 LET S=0
250 LET D=0
260 LET R=0
270 LET L=0
280 LET J=-1
290 FOR X=0 TO 1 STEP .005
300 LET F=0
310 LET P(0)=1
320 LET P(1)=X
330 LET P(2)=1.5*X+2-.5
340 FOR N=2 TO 61
350 LET P(N+1)=(X*(2*N+1)*P(N)-N*P(N-1))/(N+1)
360 NEXT N
370 FOR N=1 TO 61 STEP 2
380 LET F=F+A(N)*P(N)
390 NEXT N
400 LET S=S+F+2
410 LET D=D+F*SIN(C*.005*X)
420 LET R=R+F*SIN(C*.015*X)
430 LET L=L+F*SIN(C*.025*X)
440 LET J=J+1
450 LET Z(J)=F
460 NEXT X
470 LET Q=.1*SQR(S-.5*(Z(0)+Z(200)+Z(200)+Z(2)))
480 PRINT "NORMALIZING FACTOR="Q
490 PRINT "EIGENVALUE"
500PRINT"U(.005)="(D-.5*Z(200)*SIN(C*.005))/Z(1)/100
510PRINT"U(.015)="(R-.5*Z(200)*SIN(C*.015))/Z(3)/100
520PRINT"U(.025)="(L-.5*Z(200)*SIN(C*.025))/Z(5)/100
530 PRINT "NORMALIZED EIGENFCN"
540 PRINT "X","V(X)"
550 FOR I=0 TO 200 STEP 2
560 PRINT I/200;Z(I)/Q
570 NEXT I
580 GOTO 100
590 DATA 1114.05,.0111241
600 END

```

Remarks: This program is for the odd-index eigenelements. For the even-index cases, statement 370 becomes FOR N=0 to 60 STEP 2, 210 becomes FOR N=150 TO 1 STEP -2, and all SIN become COS.

B. PROGRAM FOR DETERMINATION OF k IN EQUATION (9) TO
A 9-PLACE ACCURACY

```

10 DIM G(100),F(100)
30 LET P 3.14159265
40 LET B (10*P)+2
55 READ L
56 LET M 1
57 FOR A=0 TO 5
60 FOR K=L-1/10+A TO L+1/10+A STEP 1/10+(A+1)
70 FOR N=3 TO 71 STEP 2
75 DEF FNR(N) (2*N+3)*(2*N+5)/((N+2)*(N+1)*B)
76 DEF FNS(N) N*(N+1)-K+(2*N*(N+1)-1)*B/(2*N-1)/(2*N+3)
77 DEF FNT(N) N*(N-1)*B/(2*N-3)/(2*N-1)
80 LET F(N) FNR(N)*FNS(N)
90 LET G(N) FNR(N)*FNT(N)
100 NEXT N
170 LET I=G(65)/(F(65)-G(67)/(F(67)-G(69)/F(69)))
180 LET H=G(59)/(F(59)-G(61)/(F(61)-G(63)/(F(63)-I)))
190 LET J=G(53)/(F(53)-G(55)/(F(55)-G(57)/(F(57)-H)))
200 LET R=G(47)/(F(47)-G(49)/(F(49)-G(51)/(F(51)-J)))
210 LET S=G(41)/(F(41)-G(43)/(F(43)-G(45)/(F(45)-R)))
220 LET T=G(35)/(F(35)-G(37)/(F(37)-G(39)/(F(39)-S)))
230 LET U=G(29)/(F(29)-G(31)/(F(31)-G(33)/(F(33)-T)))
240 LET V=G(23)/(F(23)-G(25)/(F(25)-G(27)/(F(27)-U)))
250 LET W=G(17)/(F(17)-G(19)/(F(19)-G(21)/(F(21)-V)))
260 LET X=G(11)/(F(11)-G(13)/(F(13)-G(15)/(F(15)-W)))
270 LET Y=G(5)/(F(5)-G(7)/(F(7)-G(9)/(F(9)-X)))
280 LET Z=G(3)/(F(3)-Y)-1-11.6666667/B+5.83333333*K/B
290 LET Q=ABS(2.5-Z)
300 IF Q>M THEN 340
310 IF Q<M THEN 320
320 LET M=Q
330 LET D=K
340 NEXT K
350 PRINT M; INT(L); D-INT(L)
355 LET L=D
360 NEXT A
370 PRINT
371 PRINT
380 GO TO 55
3000 DATA 139,322,502,676,846
9999 END

```

END

Remark: This program is for the odd-index cases. For the even-index cases, the arguments of F and G are even numbers and statement 280 becomes $\text{LET } Z = G(2)/(F(2)-Y) + 7.5 * K/B$.